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Cultural transmission and the evolution of gender roles☆

Victor Hiller[a,∗], Thomas Baudin[b]

[a] LEMMA, Université Paris II Panthéon-Assas, France
[b] Centre de recherche en démographie et sociétés, UC Louvain, Belgium

HIGHLIGHTS

- Cultural transmission model where parents socialize daughters and sons differently.
- Joint cultural dynamics within males and females populations.
- Application to the evolution of gender roles.
- Gender roles may differ between societies with common socio-economic features.
- Technological or institutional changes partly explain the evolution of gender roles.

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ABSTRACT

Investigating how gender roles evolve over time is crucial to understand gender inequalities and their dynamics. We propose a cultural transmission model in which parents may socialize their sons and daughters differently and we derive gender specific cultural dynamics. We show how interactions between socialization behaviors and household's economic decisions shape these dynamics and the long-run definition of gender roles. We apply our theory to cultural traits related to the prescribed role of men and women within the society. Multiple equilibria may arise and an economy may be trapped in a situation in which males and females share inegalitarian views about gender roles. Institutional changes as a greater availability of child care facilities may help to reach another equilibrium in which a large proportion of individuals hold more egalitarian views about these roles.

1. Introduction

The last decades of the twentieth century were characterized by a reduction in many aspects of gender inequalities in developed countries. During this period, the pay gap narrowed as well as the gender gap in labor force participation, women caught up with men in terms of college attendance and graduation rates. Even the concept of "gendered occupations" lost part of its meaning (see Goldin, 2006). Nevertheless, despite these evolutions and an apparent political voluntarism, gender equality is not a reality yet and some inequalities between men and women seem to persist over time. Women remain under-represented in the highest paid occupations (see Bertrand and Hallock, 2001 or Wolfers, 2006) and their life cycle earnings after having completed an MBA are significantly lower than those of men with a similar educational attainment (see Bertrand et al., 2010). In many countries the female labor force participation (FLFP hereafter) remains low, especially for mothers having young children (see, for instance, Fortin, 2005 or Del Boca and Sauer, 2009). Gemici and Wiswall (2014) also underline that the drop in the gender gap in college graduation hides important inequalities with women significantly less likely than men to major in business or science.

An important literature investigates the roots of changes and permanence in gender inequality and the role played by cultural aspects constitutes the central focus of a growing number of papers. Recent research in experimental economics highlights the existence of gender differences in preferences, concluding that women tend to adopt less risky and less competitive behaviors

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∗ Corresponding author.
E-mail addresses: victor.hiller@u-paris2.fr (V. Hiller), thomas.baudin@uclouvain.be (T. Baudin).

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than men (see Croson and Gneezy, 2009 for an extensive survey) and hypothesizing that these differences may partly explain the persistence of gender inequality.[1] Another strand of the literature convincingly relates the labor market outcomes of women to the prevailing cultural definition of gender roles within the society (Fortin, 2005, Fernández, 2007, Fernández and Fogli, 2009 or Giavazzi et al., 2013). As a striking illustration of such a link, some studies conclude that the FLFP may differ widely between populations who share a quite similar economic and institutional environment. In particular, Steinhauer (2013) shows that, within Swiss states, mothers participate more to the labor market on the French-speaking side compared to the German-speaking one despite a common institutional setting and a well integrated labor market.

At the same time, some recent findings indicate that those cultural aspects of gender inequality should be considered as endogenous. Coming back to the experimental literature, the institutional environment such as the patriarchal or matrilineal nature of a society, turns out to affect the gender gap in preferences (see Gneezy et al., 2009, Gong and Yang, 2012 or Gong et al., 2015). Fernández et al. (2004) also conclude that cultural beliefs about the appropriate role of women may be transmitted across generations by documenting that men with working mothers are less averse to have a working wife. Consequently, an increase in the participation of women to the labor market within one generation contributes to the diffusion of more tolerant norms about working wife for the next generations. Finally, Alesina et al. (2013) highlight that the gender-based division of labor, resulting from the agricultural forms practiced during pre-industrial times, affects the cultural norms about the respective role of men and women nowadays.

According to this last body of evidence, the cultural definition of gender roles would be expected to converge between two populations sharing similar economic and institutional characteristics, such as the French-speaking and German-speaking communities within Swiss states. In the present paper, we show that the contrary may be true. To that end, we propose a cultural transmission model *à la* Bisin and Verdier (2001) to investigate the evolution of gender roles over time. One new and important feature of our framework is that cultural views about gender roles may differ between men and women as parents may socialize their sons and daughters differently. Then, we show that the co-evolution of cultural distribution within populations of males and females respectively may lead to the emergence of multiple long-run equilibria. Hence, quite different definitions of gender roles might persist overtime between communities who share identical economics fundamentals. To understand the intuitions behind this result, let us briefly describe the main ingredients of our model.

In line with the existing the cultural transmission literature (see Bisin and Verdier, 2011 for a survey), parents may exert some costly effort in order to transmit their own cultural trait to their children. Since parents are altruistic, the intensity of this effort depends on the utility that they expect for their children when holding this trait. We augment Bisin and Verdier's format by allowing expected utility of children to be distinct for boys and girls respectively. As a result, parents may choose different socialization efforts for boys and girls. We are then able to propose a general set-up to study the joint cultural dynamics within both populations of males and females. Three main driving forces shape the way the joint distribution of cultural traits evolve over time. First, as usual in the cultural transmission literature, a *cultural substitution effect* arises and favors cultural diversity among each population. Second, in line with Bisin and Verdier (2000) and Bisin et al. (2004), we consider that the socialization process is more effective within homogamous couples (in which the two spouses share the same culture) than within heterogamous ones. This assumption adds a new dimension to the cultural dynamics that we call the *extensive margin of socialization*: the joint distribution of cultural traits within both populations, by determining the proportion of homogamous couples, directly affects the pattern of diffusion of those traits. Third, as parents are altruistic, their socialization choices depend on the well-being they expect for their child at the next date. In turn, as the child will be matched with a spouse of the opposite sex when adult, this expected utility is a function of two elements: (i) the expected distribution of preferences within the population of the opposite sex; and (ii) the way the well-being that the child associates to a specific cultural trait is affected by the trait of her/his mate *i.e.* the *nature of complementarity within the household*. Obviously, the nature of complementarity that prevails depends on the technological, social and institutional environment. Then, our framework constitutes a powerful tool to analyze the impact of economic and socio-institutional shocks on the evolution of gender roles.

From a theoretical point of view, we obtain some new results with respect to the existing cultural transmission literature. In particular, in the baseline model proposed by Bisin and Verdier (2001), the aforementioned paternalistic altruism induces the existence of one unique interior stable steady-state. In our framework, due to the combination between the *cultural substitution effect*, the *extensive margin of socialization* and the *nature of complementarity within the household*, an extreme steady-state, in which the cultural distribution is fully homogeneous within each population, may be reached in the long run. As previously mentioned, we also highlight the possible co-existence of multiple steady-states such that the long-run situation of the economy depends on the initial distribution of cultural traits. While this multiplicity result is not new in this strand of the literature, it is usually obtained by assuming the existence of strategic complementarity in the social environment (see Bisin and Verdier, 2011).[2] Such kind of complementarity, which arises *within* a population, is absent from our model. Instead, our multiplicity result is driven by a complementarity that could exist *between* two populations (males and females). To illustrate this and discuss the empirical relevance of our results, we apply our model to specific cultural traits.

In our application, the cultural traits we are interested in are related to the prescribed role of women in a society. Within both populations of males and females some individuals hold traditional views about gender roles (considering that men should be the breadwinner and women the homemaker), while others hold modern (or egalitarian) views about these roles. More specifically, both *traditionalist* men and women get an additional satisfaction when the wife specializes herself in family activities. Within a household, each spouse has to choose her/his degree of involvement in market activities. If the two spouses choose to be highly involved in those activities, a fixed share of income has to be dedicated to maintain the home and take care of children. This amount must be considered as a proxy for the degree of availability of child care facilities in the economy. Under those reasonable assumptions, we show that a *traditionalist equilibrium* where all individuals hold inegalitarian views about gender roles and the FLFP is low co-exists with an *egalitarian equilibrium* where a large proportion of individuals hold egalitarian views about gender roles and much more women are highly involved in market activities.

1 Some recent articles show that differences in preferences indeed matter in explaining the gender gap in fields of study (see Zafar, 2012 or Buser et al., 2014) or carrier choice (see Flory et al., 2015).

2 We say that the social environment is characterized by strategic complementarity when the incentives to transmit a trait to a child are increasing in the expected prevalence of that trait within the population of this child.

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This multiplicity result allows for a theoretical foundation of the aforementioned empirical findings by Steinhauer (2013). We also show that a greater availability of child care facilities increases the likelihood for an economy to end up in the *modern equilibrium* and, by the way, rises the expected participation of women to the labor market. The later result is backed up by the empirical studies of Del Boca and Sauer (2009) or Lauer and Weber (2003).

The present article relates to research on the endogenous evolution of gender roles. Hazan and Maoz (2002), Fernández et al. (2004) or Hiller (2014) consider that cultural norms regarding those roles are directly shaped by the participation of women to the labor market. In turn, those norms affect the FLFP by determining the magnitude of the social stigma against working women (Hazan and Maoz, 2002; Fernández et al., 2004) or the weight that parents give to the education of their daughters (Hiller, 2014). Two recent papers (Fogli and Veldkamp, 2011; Fernández, 2013) make a step further by explicitly modeling cultural evolutions as arising from a process of learning: Bayesian updating allows women to learn the impact that their decision to work on the market have on their children or their marriage. While in this *cultural norms literature* gender roles are defined at the household level, we consider that cultural views about those roles may differ between males and females. Fernández and Fogli (2009) document that it is indeed the case and, as far as we know, this possibility has not heretofore been investigated. We also depart from the *cultural norms literature* in the way we introduce the evolution of gender roles. In our model, cultural traits are intentionally transmitted by parents to their children in line with the *cultural transmission literature* initiated by Bisin and Verdier (2001). Hence, our framework may be useful to analyze the dynamics of gender differences in preferences up to the evolution of gender roles. With regard to the cultural transmission literature, we are close to Escriche (2007) and Escriche et al. (2004) who are interested in the evolution of family-related preferences among women. Our application deals with the dynamics of a quite similar cultural trait. However, we endogenize the distribution of traits within both the populations of males and females, while Escriche (2007) and Escriche et al. (2004) consider men's preferences as exogenous. We also enrich their approach by offering a central role to interactions between spouses within the household and by taking into account the *extensive margin of socialization*, two dimensions that are absent from their work. More broadly, we propose a framework in which the distribution of preferences evolves jointly within two populations. To our knowledge, it is a first attempt to take into account the fact that the evolution of traits within one population may also depend on the distribution of traits within another one.

The rest of the paper is organized as follows. Section 2 exposes the model and Section 3 its dynamics properties. In these two sections, we keep a general framework without specifying particular cultural traits. Our application is proposed in Section 4. Section 5 concludes. Main proofs are gathered in the Appendix.

2. The model

We consider an economy populated by overlapping generations of men and women. At each date t, the populations of males (m) and females (f) are equally sized. Each individual lives for two periods: childhood and adulthood. Within each population of adults two cultural traits $\{a, b\}$ co-exist. Each trait is associated with specific preferences (represented by a utility function) that may differ between gender. Those traits are acquired during childhood. Within population $s \in \{f, m\}$, the proportion of adults with trait a at date t is denoted q_t^s. During adulthood, each male is randomly matched with a female to form a family. Each family has two children, one boy and one girl, and parents may spend efforts in order to shape their children's culture.

2.1. Cultural transmission mechanisms

As it will be clearly exposed soon in the next section, parents are biased towards their own cultural trait to the extent that they display a taste for transmitting this trait. They also have a technology to transmit their own trait. In line with Bisin and Verdier (2000) and Bisin et al. (2004), we assume that homogamous couples have a more efficient technology compared to heterogamous ones. Let us expose more formally how the transmission process differs within homogamous and heterogamous families.

Socialization technology within homogamous families. At date t, a child s belonging to a homogamous family of type i (where both parents have the same trait i) is directly socialized along this trait with a probability denoted $\tau_t^{s,i}$. This probability exactly equals the socialization effort chosen by parents. Note that we depart from the standard cultural transmission model proposed by Bisin and Verdier (2001) by assuming that socialization efforts may differ between daughters and sons. With probability $1-\tau_t^{s,i}$ parents fail to directly transmit their own trait i and the child adopts the cultural trait of a *role model* randomly picked within the population of adults of the same gender (she/he adopts the trait a with probability q_t^s andthe trait b with probability $1-q_t^s$).[3]

Socialization technology within heterogamous families. As heterogamous couples do not have a clear cultural model to transmit, we follow Bisin and Verdier (2000) by assuming that those families are not able to directly socialize their children.[4] Hence, a child s from a heterogamous family is directly influenced by the society at large: she/he adopts the trait a with probability q_t^s and the trait b with probability $1-q_t^s$.

Based on the description of the socialization technology, we derive the following expressions for the probability $P_t^{s,i}$ that a child of gender s and born at t within a homogamous family of type $i \in \{a, b\}$ adopts the cultural trait of her/his parents:

$$P_t^{s,a}(\tau_t^{s,a}, q_t^s) = \tau_t^{s,a} + (1-\tau_t^{s,a})q_t^s \tag{1}$$

$$P_t^{s,b}(\tau_t^{s,b}, q_t^s) = \tau_t^{s,b} + (1-\tau_t^{s,b})(1-q_t^s). \tag{2}$$

We also derive the following expression for the law of motion of q_t^s:

$$q_{t+1}^s = q_t^s q_t^{-s} P_t^{s,a} + (1-q_t^s)(1-q_t^{-s})(1-P_t^{s,b}) + [q_t^s(1-q_t^{-s}) + q_t^{-s}(1-q_t^s)]q_t^s. \tag{3}$$

In the right hand side of (3), the first term is the proportion of child s born within a homogamous family of type a times the probability $P_t^{s,a}$ that those children adopt the trait a, the second term is the proportion of child s born within a homogamous family of type b times the probability $1-P_t^{s,b}$ that those children adopt the trait a and the third term is the proportion of child s born within a heterogamous family times the probability that those children adopt the trait a. This probability simply equals the proportion of a-type role models within the population s in t (q_t^s).

3 Girls are influenced only by female role models, while boys are influenced only by male role models. This *same-gender role model assumption* greatly simplifies the analysis of the dynamics. Several studies in sociology and psychology find that females are indeed much more likely to choose female role models, while the reverse is true for males (see, for instance, Gilbert et al., 1983, Wohlford et al., 2004, or Bricheno and Thornton, 2007).

4 This assumption captures, in a stylized way, the fact that cultural transmission is less effective when parents do not share common values or preferences (see Bisin and Verdier, 2000 and references therein for further discussion of this assumption and Bisin et al. (2004) for an alternative formalization of a similar argument).

2.2. Household decisions

The spouses within a household face two types of decisions: socialization and economic decisions. As usual in the cultural transmission literature (see Bisin and Verdier, 2011) we assume that: (i) the utility derived from socialization choices and the utility derived form economic decisions enter as separate arguments in the utility function; and that (ii) the costs of socialization are expressed in terms of utility and thus do not enter the household's budget constraint. Under those assumptions economic and socialization decisions can be analyzed separately. We will successively deal with each kind of decisions.

2.2.1. Economic decisions

Economic decisions are captured abstractly: a spouse s who holds the trait $i \in \{a, b\}$ and lives with a partner of type $j \in \{a, b\}$ derives a utility $v^{s,i}(x^{s,i}(j))$ from the choice $x^{s,i}(j) \in X^s$. The vector $x^{s,i}(j)$ contains any kind of economic decisions which are not related to socialization like consumption, labor supply, leisure time or occupational choices. Economic decisions are the result of a joint decision process within the household.[5] Choices of a couple $\{i, j\}$, stemming from this process, are denoted $(\tilde{x}^{s,i}(j), \tilde{x}^{-s,j}(i))$. Moreover, we define:

$$u^{s,i}(j) \equiv v^{s,i}(\tilde{x}^{s,i}(j)) \tag{4}$$

where $u^{s,i}(j)$ is the indirect utility that an adult s with cultural trait i derives from sharing her/his life with a j-type mate. Note that $u^{m,i}(j)$ and $u^{f,i}(j)$ are assumed to be independent of the distribution of preferences and may differ in various way, for instance: a same cultural trait may be associated with different utility functions for men and women ($v^{m,i}(.) \neq v^{f,i}(.)$), the set of available opportunities may differ between gender ($X^m \neq X^f$), the existence of a gender wage gap may translate into gender differences in labor force participation, when the decision process is cooperative the bargaining power may be unequally distributed between spouses, etc.

Based on these indirect utilities we define:

$$A^s \equiv u^{s,a}(a) - u^{s,b}(a) \quad \text{and} \quad B^s \equiv u^{s,b}(b) - u^{s,a}(b). \tag{5}$$

Parameter A^s (resp. B^s) measures the utility gain or loss for an individual s matched with a person of type a (resp. b) when being a (resp. b) too.

Definition 1. We say that, within population s:

(i) The trait a dominates the trait b if: $A^s \geq 0$ and $B^s \leq 0$ with at least one strict inequality.
(ii) The trait b dominates the trait a if: $A^s \leq 0$ and $B^s \geq 0$ with at least one strict inequality.
(iii) There exists a symmetric complementarity between cultural traits if: $A^s > 0$ and $B^s > 0$.
(iv) There exists an asymmetric complementarity between cultural traits if: $A^s < 0$ and $B^s < 0$.

Considering the population of women, we say that there is a symmetric complementarity when a female will always be better off if she exhibits the same cultural trait than her partner, whatever this trait. Conversely, if women's cultural traits display asymmetric complementarity, a wife will be better off if her trait differs from her husband's one, whatever this trait. Note that cultural traits do not necessarily exhibit the same kind of complementarity among men on the one hand and women on the other.

2.2.2. Socialization decisions

Parents care about the cultural traits of their children and, when both parents share the same trait, they can exert efforts in order to transmit it (see Section 2.1). Those efforts are costly and socialization costs are assumed to be quadratic: $C(\tau_t^{s,i}) = (\tau_t^{s,i})^2/2k$, where k accounts for the efficiency of the socialization technology. For ease of simplicity we assume that both parents bear these costs. Socialization efforts are motivated by altruism. However this altruism is imperfect since parents are biased towards their own culture. We denote $\hat{V}_t^{s,ij}$ the utility that a parent of type i attributes to her/his child s of trait j. When the trait of the child coincides to the parental one ($j = i$), this utility writes as:

$$\hat{V}_t^{s,ii}(\hat{q}_{t+1}^{-s}) = \underbrace{\hat{q}_{t+1}^{-s} u^{s,i}(a) + (1 - \hat{q}_{t+1}^{-s}) u^{s,i}(b)}_{(A)} + \underbrace{I}_{(B)} \tag{6}$$

with $\hat{q}_{t+1}^{s}$ the expectation of the proportion of a-individuals within population s at date $t + 1$. Expression (6) may be decomposed into two elements:

(A) A *Parental altruism effect* which corresponds to the expected utility of a child s when being of type i in $t + 1$.
(B) A *Cultural intolerance effect* which takes the form of an extra utility when the child belongs to the same cultural group as the parent does. The parameter $I > 0$ measures the magnitude of this effect, reflecting the degree of cultural intolerance of parents with respect to deviation from their own culture.[6]

Obviously, the parameter I does not appear in the expression of $\hat{V}_t^{s,ij}$ when $i \neq j$, which simply equals the expected utility of a child s who holds trait j in $t + 1$:

$$\hat{V}_t^{s,ij}(\hat{q}_{t+1}^{-s}) = \hat{q}_{t+1}^{-s} u^{s,j}(a) + (1 - \hat{q}_{t+1}^{-s}) u^{s,j}(b). \tag{7}$$

Within a homogamous family of type i, the two parents bear socialization costs and share a common value of $\hat{V}_t^{s,ij}$. Thus, they agree on socialization decision: a level of effort $\tau_t^{s,i}$, for each child $s \in \{f, m\}$, is chosen in order to solve the following maximization problem

$$\max_{\tau_t^{m,i}, \tau_t^{f,i}} \sum_{s \in \{m,f\}} \Big\{ P_t^{s,i}(\tau_t^{s,i}, q_t^s) \hat{V}_t^{s,ii}(\hat{q}_{t+1}^{-s}) + (1 - P_t^{s,i}(\tau_t^{s,i}, q_t^s)) \hat{V}_t^{s,ij}(\hat{q}_{t+1}^{-s}) - C(\tau_t^{s,i}) \Big\}. \tag{8}$$

The following condition enables to avoid corner solutions in socialization decisions:

Assumption 1. $I > \max\{|A^s|, |B^s|\}$ and $k < \min\left\{\frac{1}{I+A^s}, \frac{1}{I+B^s}\right\}$ for $s \in \{m, f\}$.

Moreover, we restrict our attention to the case where expectations are myopic so that

Assumption 2. $\hat{q}_{t+1}^s = q_t^s$.

Then, when putting expressions (1), (2), (6) and (7) into the socialization problem (8) and solving this program, we obtain:

$$\tau_t^{s,a}(q_t^s, q_t^{-s}) = k(1 - q_t^s) \Delta V_t^{s,a}(q_t^{-s}) \tag{9}$$

$$\tau_t^{s,b}(q_t^s, q_t^{-s}) = k q_t^s \Delta V_t^{s,b}(q_t^{-s}) \tag{10}$$

with

$$\Delta V_t^{s,a}(q_t^{-s}) = \hat{V}_t^{s,aa}(q_t^{-s}) - \hat{V}_t^{s,ab}(q_t^{-s}) = I + \Psi^s(q_t^{-s}) \tag{11}$$

$$\Delta V_t^{s,b}(q_t^{-s}) = \hat{V}_t^{s,bb}(q_t^{-s}) - \hat{V}_t^{s,ba}(q_t^{-s}) = I - \Psi^s(q_t^{-s}) \tag{12}$$

5 Let us remain quite general with regard to this process. In particular it could be a cooperative process or a non-cooperative one.

6 Cultural intolerance is assumed to depend neither on the sex of the child nor on the cultural type of parents.

and

$$\Psi^s(q_t^{-s}) \equiv q_t^{-s}A^s - (1-q_t^{-s})B^s. \tag{13}$$

Assumption 1 ensures that both $\tau_t^{s,a}(q_t^s, q_t^{-s})$ and $\tau_t^{s,b}(q_t^s, q_t^{-s})$ belong to the interval $(0, 1)$ for all $(q_t^s, q_t^{-s}) \in (0, 1)^2$. Then, according to Eqs. (11) and (12), whatever their own type, parents' incentives to transmit their culture is increasing in the degree of intolerance I. The function $\Psi^s(q_t^{-s})$ accounts for the altruistic component of socialization motives. More precisely, $\Psi^s(q_t^{-s})$ equals the expected utility gain or loss for a child s when being a rather than b. It obviously depends on the expected distribution of preferences within the population $-s$ (which equals q_t^{-s} when expectations are myopic) and on the nature of complementarity within the household (parameters A^s and B^s).

3. The cultural dynamics

3.1. The dynamical system

Using the transition probabilities (9) and (10) into Eq. (3) that describes the law of motion of q_t^s, and after simple algebra, we obtain the following bi-dimensional first-order dynamical system that governs the joint evolution of cultural traits within the two populations:

$$\begin{cases} q_{t+1}^f = q_t^f + q_t^f(1-q_t^f)[q_t^m \tau_t^{f,a}(q_t^m, q_t^f) \\ \qquad - (1-q_t^m)\tau_t^{f,b}(q_t^m, q_t^f)] \\ q_{t+1}^m = q_t^m + q_t^m(1-q_t^m)[q_t^f \tau_t^{m,a}(q_t^m, q_t^f) \\ \qquad - (1-q_t^f)\tau_t^{m,b}(q_t^m, q_t^f)]. \end{cases} \tag{14}$$

Before we provide a formal analysis of this system, let us discuss the way q_t^s varies overtime. Through Eqs. (11), (12) and (14), the direction of changes of q_t^s (for a given value of q_t^{-s}) may be expressed as:

$$\Delta q_t^s \equiv q_{t+1}^s - q_t^s \overset{\text{sign}}{=} \underbrace{q_t^{-s}}_{(1)} \underbrace{(1-q_t^s)}_{(2)} \underbrace{\Delta V_t^{s,a}(q_t^{-s})}_{(3)} - \underbrace{(1-q_t^{-s})}_{(1)} \underbrace{q_t^s}_{(2)} \underbrace{\Delta V_t^{s,b}(q_t^{-s})}_{(3)}. \tag{15}$$

As highlighted in expression (15), this direction of changes may be decomposed in three arguments that should be interpreted in the following way:

(1) *Extensive margin of socialization*: As only homogamous couples prove to be able to directly transmit their culture, the number of children socialized along the trait a (resp. b) is increasing in the proportion of homogamous couples of type a (resp. b). For a given value of q_t^s, this proportion is positively (resp. negatively) related to q_t^{-s}.
(2) *Cultural substitution*: Since direct socialization (by parents) and indirect socialization (by the society at large) are substitutes, the higher is q_t^s, the higher (resp. lower) is the probability that a child adopts the trait a (resp. b) if parental socialization fails. Hence, transmission efforts by homogamous couples of type a (resp. b) are decreasing (resp. increasing) in q_t^s.
(3) *Nature of the complementarity within the household:* Because parents are altruistic, their socialization choices depend on the well-being they expect for their child which is a function of the distribution of traits within the population of the opposite sex q_t^{-s} (see expressions (11)–(13)). The impact of q_t^{-s} on the incentives to transmit the trait a rather than b depends on the nature of complementarity that arises within the household (parameters A^s and B^s).[7]

Using Eqs. (11) and (12), expression (15) may be rewritten as follows:

$$\Delta q_t^s \overset{\text{sign}}{=} \underbrace{(q_t^{-s} - q_t^s)I}_{(i)} + \underbrace{[q_t^s(1-q_t^{-s}) + (1-q_t^s)q_t^{-s}]\Psi^s(q_t^{-s})}_{(ii)}. \tag{16}$$

Expression (16) allows us to shed a new light on the main forces behind the diffusion of the trait a over time. In particular, two prominent effects emanate from the combination of the three aforementioned arguments:

(i) A *cultural intolerance effect*. For a given value of I, it is all the more strong that the difference $(q_t^{-s} - q_t^s)$, reflecting the relative strength of the extensive margin *vs.* cultural substitution effects, is large. Indeed, a high value of q_t^{-s} reinforces the extensive margin effect by increasing the proportion of homogamous couples of type a; while a high value of q_t^s accentuates the cultural substitution effect by reducing the incentives to transmit the trait a among those couples. Thus, when cultural traits tend to be equally distributed between both populations (q_t^{-s} close to q_t^s) the impact of cultural intolerance vanishes as these two effects compensate each other.
(ii) An *altruistic motives effect*. As the altruistic component of preferences does not depend on the cultural trait of parents, the impact of the altruistic motives effect only depends on the sign of $\Psi^s(q_t^{-s})$.[8]

The cultural dynamics of the society (Eq. (14)) may be intuitively represented by a phase diagram. In order to draw it, we have to depict the ***ff*** and ***mm*** loci where q_t^f and q_t^m are respectively at a steady-state. This is what we do in the next section.

3.2. The *ff* and *mm* loci

The ***ff*** (resp. ***mm***) locus is defined as the set of pairs (q_t^f, q_t^m) such that q_t^f (resp. q_t^m) is constant: $\boldsymbol{ff} = \{(q_t^f, q_t^m) \in [0, 1]^2 : \Delta q_t^f = 0\}$ and $\boldsymbol{mm} = \{(q_t^f, q_t^m) \in [0, 1]^2 : \Delta q_t^m = 0\}$. It directly follows from (15) that the condition $\Delta q_t^s = 0$ is equivalent to $q_t^s = Q^s(q_t^{-s})$ with:

$$Q^s(q_t^{-s}) = \frac{q_t^{-s}\Delta V_t^{s,a}(q_t^{-s})}{q_t^{-s}\Delta V_t^{s,a}(q_t^{-s}) + (1-q_t^{-s})\Delta V_t^{s,b}(q_t^{-s})}. \tag{17}$$

Thus, in the plane (q_t^f, q_t^m), the ***mm*** and ***ff*** loci are respectively given by $q_t^m = Q^m(q_t^f)$ and $q_t^m = \chi^f(q_t^f)$ with $\chi^f(q_t^f)$ the inverse function of $Q^f(q_t^m)$. The precise shape of the ***mm*** and ***ff*** loci are detailed in Appendix A (Lemmas 3–5). We also establish in the Appendix that each locus is globally stable (Lemma 7).[9]

Based on these properties we are able to illustrate how the nature of complementarity within the household affects the shape of the ***mm*** locus (see Fig. 1).[10] We first note that, in both the symmetric complementarity and the asymmetric

[7] Formally, $\frac{\partial \Delta V_t^{s,a}(q_t^{-s})}{\partial q_t^{-s}} = -\frac{\partial \Delta V_t^{s,b}(q_t^{-s})}{\partial q_t^{-s}} = \frac{\partial \Psi^s(q_t^{-s})}{\partial q_t^{-s}} = A^s + B^s$.

[8] The term $[q_t^s(1-q_t^{-s}) + (1-q_t^{-s})q_t^s]$ is positive for all $(q_t^s, q_t^{-s}) \in (0, 1)^2$. Nevertheless, this term captures the magnitude of the altruistic motives effect that is a function of both the proportion of homogamous couples (extensive margin) and the intensity of direct socialization among those couples (cultural substitution).

[9] This stability result recalls a well-known property of endogenous cultural transmission models directly linked to cultural substitution. Indeed, and as previously discussed, due to cultural substitution, the larger q_t^s is the lower the incentives to transmit the trait a to a child s are. This force tends to attract q_t^m (resp. q_t^f) to the ***mm*** (resp. ***ff***) locus.

[10] The shape of the ***ff*** locus may be deduced by symmetry. In Fig. 1, the ***mm*** locus is in bold line and the 45° line in thin line. The motion arrows show how q_t^m evolves off the loci. Their directions are directly deduced from the stability result in Lemma 7.

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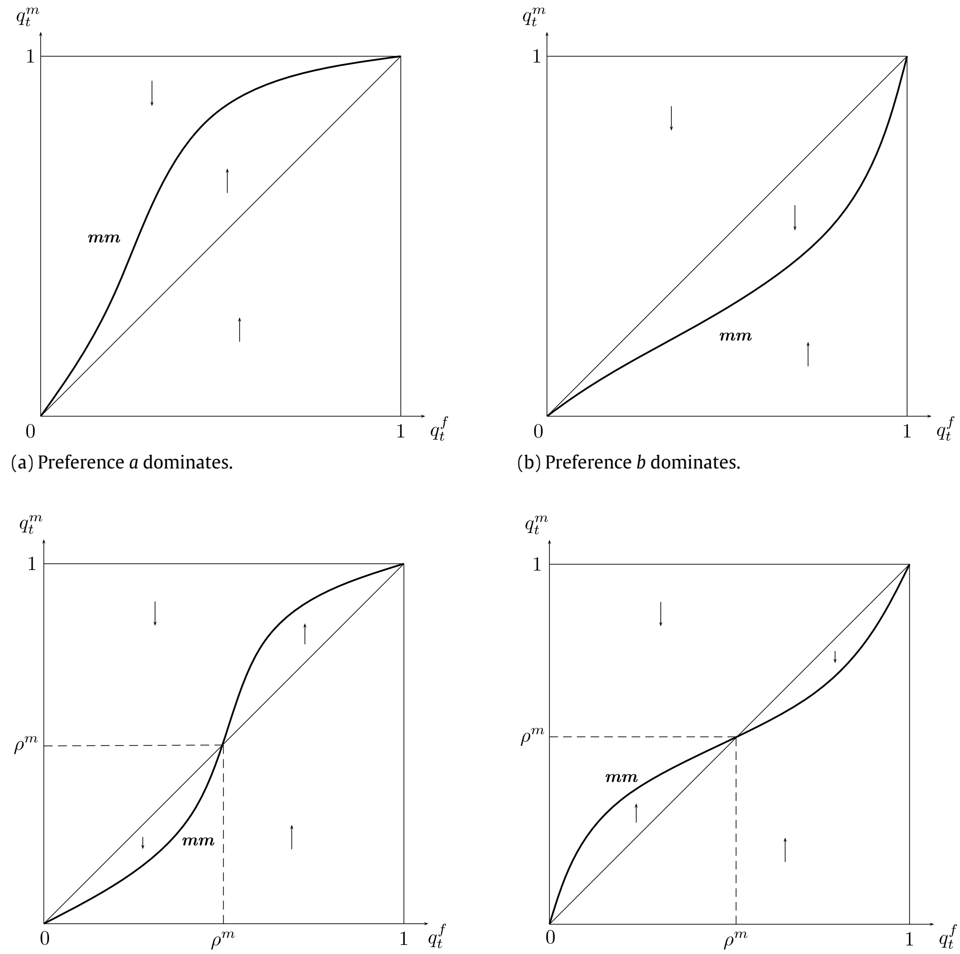


(a) Preference a dominates.
(b) Preference b dominates.
(c) Symmetric complementarity.
(d) Asymmetric complementarity.

Fig. 1. The ***mm*** locus.

complementarity cases, the ***mm*** (resp. ***ff***) locus crosses the 45° line in $q_t^f = \rho^m$ (resp. $q_t^m = \rho^f$) with

$$\rho^s = \frac{B^s}{A^s + B^s}.$$

Indeed, $\Psi^s(\rho^s) = 0$ such that, when $q_t^{-s} = \rho^s$, $\Delta q_t^s = 0$ as soon as $q_t^m = q_t^f$ (see Eq. (16)).

The shape of the ***mm*** locus reflects the tensions existing between the aforementioned *altruistic motives* and the *cultural intolerance* effects. On the one hand, the *altruistic motives effect* fosters the homogenization of the cultural distribution. For instance, when preference a dominates, $\Psi^m(q_t^f)$ is always positive such that, in the absence of cultural intolerance (*i.e.* when $I = 0$) q_t^m should converge towards 1. When considering the case of symmetric complementarity, $\Psi^m(q_t^f) \gtreqless 0$ when $q_t^f \gtreqless \rho^m$ such that, in the absence of cultural intolerance, q_t^m should converge towards 0 (resp. 1) when q_t^f is smaller (resp. larger) than ρ^m. On the other hand, the *cultural intolerance effect* favors cultural diversity. More precisely, in the absence of altruistic motives (*i.e.* when $\Psi^m(q_t^f) = 0$) the ***mm*** locus should be given by 45° line. Indeed, when $q_t^f = q_t^m$, the extensive margin effect exactly offsets the cultural substitution effect such that the impact of cultural intolerance falls to zero.

Let us consider once again the case where the trait a dominates (Fig. 1(a)), since the *altruistic motives effect* always favors the diffusion of this trait, the ***mm*** locus stands to be above the 45° line. Still in that case, when (q_t^f, q_t^m) is located below the 45° line the two effects go in the same direction and q_t^m increases overtime. In the area located between the ***mm*** locus and the 45° line, q_t^m increases despite the fact that it is higher than q_t^f then we say that *altruistic motives dominate cultural intolerance*. Finally, when (q_t^f, q_t^m) is located above the ***mm*** locus, q_t^m decreases despite the fact that trait a dominates, then we say that *cultural intolerance dominates altruistic motives*.

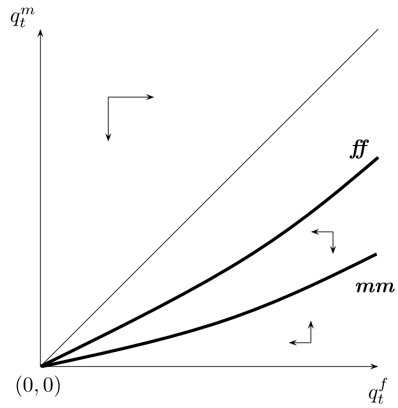
3.3. The steady-states

The steady-states of the cultural dynamics are defined as those pairs (q_t^f, q_t^m) such that $\Delta q_t^f = 0$ and $\Delta q_t^m = 0$. A simple inspection of (14) allows us to conclude that the set of steady-states is constituted by (0, 0), (1, 1), (0, 1), (1, 0) as well as the crossing points between the ***ff*** and ***mm*** loci (the *interior equilibria* hereafter). The following lemma lists the stability properties of the corner steady-states (0, 0), (1, 1), (0, 1) and (1, 0).

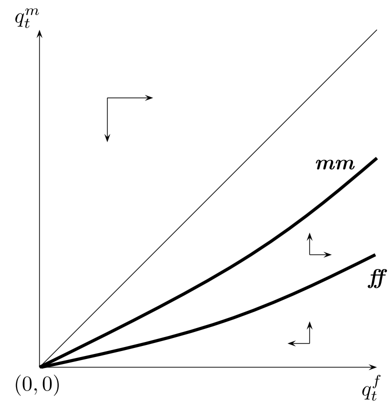
Lemma 1. *Under Assumptions 1 and 2:*

- *The steady-states* (0, 1) *and* (1, 0) *are unstable.*
- *The steady-state* (0, 0) *is unstable if* $B^m < -B^f$ *and locally stable if* $B^m > -B^f$.
- *The steady-state* (1, 1) *is unstable if* $A^m < -A^f$ *and locally stable if* $A^m > -A^f$.

Proof. These results may be directly deduced from (i) the properties of the slope of ***ff*** and ***mm*** loci in 0 and 1 (see Lemma 5) and (ii) the global stability of ***ff*** and ***mm*** loci (see Lemma 7). ■



(a) $B^m > -B^f$.



(b) $B^m < -B^f$.

Fig. 2. Local stability of the (0, 0) equilibrium when $B^m > 0$ and $B^f < 0$.

A deeper look at Eq. (16) enables to figure out the stability property of the inegalitarian steady-states (0, 1) and (1, 0). In the neighborhood of the steady-state (0, 1) preferences are quite unequally distributed between gender since almost all women are *b* while almost all men are *a*. Consequently, the weight of the parameter *I* in the cultural dynamics (16) is heavy (equal to 1 in the expression of Δq_t^f and to -1 in the expression of Δq_t^m). Moreover, under Assumption 1, $I+\Psi^f(q_t^m)$ is always positive while $-I+\Psi^m(q_t^f)$ is always negative. Thus, the cultural intolerance effect dominates altruistic motives and q_t^m (resp. q_t^f) tends to decrease (resp. increase). A symmetric reasoning applies for the steady-state (1, 0). Let us underline that this result crucially relies on Assumption 1 according to which cultural intolerance is high enough to ensure that parents always aim at transmitting their own culture. This condition plays the role of the imperfect empathy assumption usually adopted in cultural transmission models *à la* Bisin and Verdier.[11]

Let us turn now to the stability properties of the egalitarian steady-states. As claimed in Lemma 1, the comparison between A^m and $-A^f$ on the one hand and B^m and $-B^f$ on the other hand is crucial. To see this, let us focus on the (0, 0) equilibrium. In the neighborhood of this steady-state almost all men and all women are *b* such that $\Psi^f(q_t^m) \approx -B^f$ and $\Psi^m(q_t^f) \approx -B^m$. Consequently, in the symmetric complementarity case ($B^f > 0$ and $B^m > 0$), both $\Psi^f(q_t^m)$ and $\Psi^m(q_t^f)$ are negative and the steady-state (0, 0) is stable. Indeed, in that case, individuals are always better off when sharing the culture of their mate. Hence, the incentives to transmit trait *a* are mitigated by the low values of q_t^f and q_t^m. The reverse is true in the asymmetric complementarity case such that (0, 0) becomes unstable.

Let us now consider a situation where $B^m > 0$ and $B^f < 0$. Then, in the neighborhood of (0, 0), altruistic motives support the diffusion of the trait *b* for girls and *a* for boys and both the ***mm*** and the ***ff*** loci are located below the 45° line as illustrated in Fig. 2. As previously discussed, if q_t^f and q_t^m are too far from each other, the cultural intolerance effect always dominates. In particular, when (q_t^m, q_t^f) is located above (resp. below) the two loci, q_t^m decreases (resp. increases) while q_t^f increases (resp. decreases) overtime such that (q_t^m, q_t^f) reaches the area located between the ***mm*** and the ***ff*** loci. From then on, the way the joint distribution of traits evolves fundamentally depends on the comparison between B^m and $-B^f$ that is related to the intensity of incentives that parents have to socialize their sons and daughters respectively. When, $B^m > -B^f$ (implying $|\Psi^m(q_t^f)| > |\Psi^f(q_t^m)|$) the relative incentives, driven by altruistic motives, that parents have to transmit the trait *b* to boys overcomes the relative incentives they have to transmit the trait *a* to girls (graphically, the ***mm*** locus is located below the ***ff*** one). Hence, between the two loci, altruistic motives dominate cultural intolerance for boys while the reverse is true for girls and both q_t^m and q_t^f decrease overtime (Fig. 2(a)). A symmetric reasoning applies for $B^m < -B^f$ (Fig. 2(b)).

In the next section, we analyze how the exact number of interior steady-states and their stability properties are affected by the sign of the parameters A^s and B^s. Then, we can describe how the long-run behavior of the economy depends on the nature of complementarity within the household.

3.4. The distribution of cultural traits in the long run

The same trait dominates within both populations. The following proposition describes the long-run distribution of cultural traits when either the trait *a* or *b* dominates within both populations.

Proposition 1. *Under Assumptions 1 and 2:*

- *When the trait a dominates within both populations* (1, 1) *is the unique globally stable steady-state while there are three unstable steady-states:* (0, 0), (0, 1) *and* (1, 0).
- *When the trait b dominates within both populations* (0, 0) *is the unique globally stable steady-state while there are three unstable steady-states:* (1, 1), (0, 1) *and* (1, 0).

The first point claimed in Proposition 1 is illustrated by the phase diagram in Fig. 3. When the trait *a* dominates within both populations, the cultural dynamics lead to a complete homogenization of the society such that trait *b* disappears in the long run.

To figure out this pattern, let us refer to our description of the dynamical system in Section 3.1. Recall that the cultural intolerance effect moves q_t^f and q_t^m towards the center of the state space. Moreover, as q_t^f and q_t^m get closer, the relative role played by altruistic motives in the diffusion of both traits is reinforced. Finally, as the trait *a* dominates within both populations, the later motives favor the diffusion of the trait *a* (both $\Psi^f(q_t^m)$ and $\Psi^m(q_t^f)$ are always positive). Let us underline that this result of cultural homogenization is driven by the interactions existing between socialization choices within the two populations. Indeed, as claimed in Lemma 7 and as usual in the cultural transmission

[11] Baudin and Hiller (2014) show that when altruism is perfect ($I = 0$) the inegalitarian steady-states (0, 1) and (1, 0) may be locally stable.

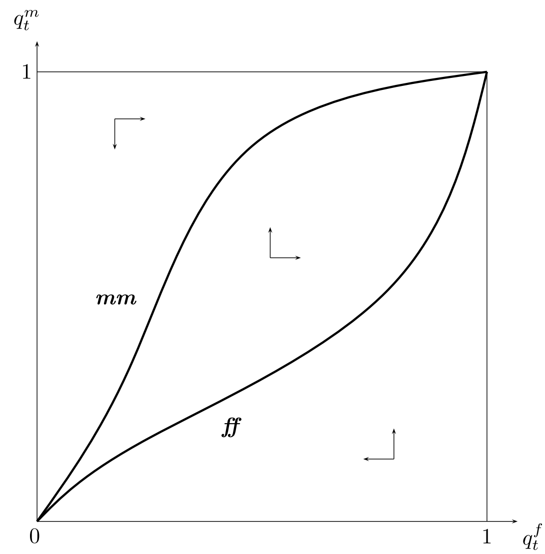


Fig. 3. The phase diagram with $A^f > 0$, $B^f < 0$, $A^m > 0$ and $B^m < 0$.

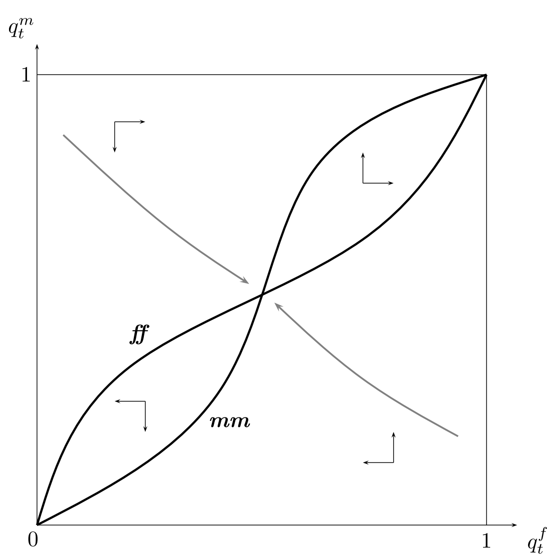


Fig. 4. The phase diagram with $A^f > 0$, $B^f > 0$, $A^m > 0$ and $B^m > 0$.

literature, each dynamics analyzed separately is characterized by a unique and globally stable interior steady-state.

Symmetric complementarity within both populations. The following proposition describes how the dynamics of cultural traits behave in the symmetric complementarity case.

Proposition 2. *Under Assumptions 1 and 2 and when there exists symmetric complementarity within both populations, two locally stable steady-states* $(0, 0)$ *and* $(1, 1)$ *co-exist with three unstable steady-states:* $(0, 1)$, $(1, 0)$ *and one interior equilibrium.*

This result is illustrated by the phase diagram in Fig. 4. In this diagram, we have also drawn the saddle paths converging towards the unstable interior steady-state (gray arrows). These saddle paths share the state space into the basin of attraction of $(0, 0)$ (below the saddle paths) and the basin of attraction of $(1, 1)$ (above the saddle paths).

Once again, cultural substitution moves q_t^f and q_t^m towards the center of the state space such that the role played by cultural intolerance is downsized. When coming to altruistic motives, due to the existence of a symmetric complementarity within the household, $\Psi^m(q_t^f)$ is positive if and only if q_t^f is large enough (larger than ρ^m) and $\Psi^f(q_t^m)$ is positive if and only if q_t^m is large enough (larger than ρ^f). Hence, when both q_t^f and q_t^m are relatively low (resp. high), these two proportions tend to diminish (resp. increase) overtime such that $(0, 0)$ and $(1, 1)$ are locally stable.

As in the previous case, cultural homogeneity prevails in the long run. Moreover, in the symmetric complementarity case, multiple equilibria arise. This result is also driven by the existing interactions between socialization choices for girls and boys[12]: the diffusion of the trait *a* within one population reinforces the incentives to transmit this trait within the other population.

Multiple equilibria imply that the long-run situation reached by the economy crucially depends on the initial distribution of cultural traits. The following proposition establishes how some changes in the parameter values do affect the likelihood to end up in one steady-state rather than the other.

Proposition 3. *Under Assumptions 1 and 2, the unstable interior steady-state is:*

- *shifted up to the right when either, B^f increases, A^f decreases, B^m increases or A^m decreases;*
- *shifted down to the left when either, B^f decreases, A^f increases, B^m decreases or A^m increases.*

Proof. These results directly follow from comparative statics results of Lemma 6. ■

As claimed in Proposition 3, when the relative incentives to transmit the trait *a* rather than *b* within one population grow, the basin of attraction of the $(1, 1)$ steady-state enlarges. Hence, even if preferences of men hold unchanged, an increase in A^f rises the likelihood for the trait *b* to disappear within the population of females, but also within the population of males.

Asymmetric complementarity within both populations. The following proposition deals with dynamical implications of asymmetric complementarity.

Proposition 4. *Under Assumptions 1 and 2 and when there exists asymmetric complementarity within both populations, one unique interior globally stable steady-state co-exists with four unstable steady-states:* $(0, 0)$, $(1, 1)$, $(0, 1)$ *and* $(1, 0)$.

This claim is illustrated by the phase diagram in Fig. 5. Here, incentives to transmit one trait are decreasing in the prevalence of this trait within the population of the opposite sex. More specifically, $\Psi^m(q_t^f)$ is positive if and only if q_t^f is small enough (smaller than ρ^m) and $\Psi^f(q_t^m)$ is positive if and only if q_t^m is small enough (smaller than ρ^f). Hence, in the long run, the two cultural traits co-exist within both populations.

Denoting $(\bar{q}^f, \bar{q}^m)$ the interior stable steady-state, we can claim that:

Proposition 5. *Under Assumptions 1 and 2,* $(\bar{q}^f, \bar{q}^m)$ *is:*

- *shifted up to the right when either, B^f decreases, A^f increases, B^m decreases or A^m increases;*
- *shifted down to the left when either, B^f increases, A^f decreases, B^m increases or A^m decreases.*

Moreover:

- *If $\rho^m > \rho^f$: $\rho^f < \bar{q}^f < \bar{q}^m < \rho^m$.*
- *If $\rho^m = \rho^f = \rho$: $\bar{q}^f = \bar{q}^m = \rho$.*
- *If $\rho^m < \rho^f$: $\rho^f > \bar{q}^f > \bar{q}^m > \rho^m$.*

12 Since indirect utilities $u^{s,i}(j)$ are independent on the distribution of traits, the cultural dynamics within population *s* (considering q_t^{-s} as given) exhibits a unique globally stable interior steady-state.

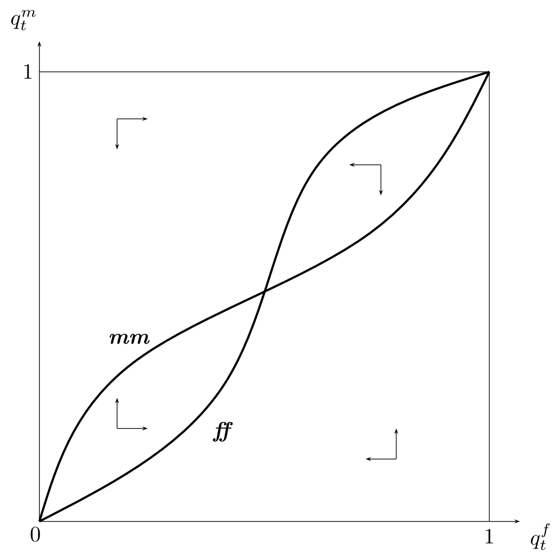


Fig. 5. The phase diagram with $A^f < 0, B^f < 0, A^m < 0$ and $B^m < 0$.

The first part of Proposition 5 establishes that, at the steady-state, when relative incentives to transmit the trait a rather than b within one population grow, the prevalence of the trait a increases within both populations. One of the main lesson to be learned from the second point of Proposition 5 is that, if there is no ex-ante difference between men and women ($A^f = A^m$ and $B^f = B^m$), in the long-run, cultural traits tend to be identically distributed in the two populations. Put differently, nurture complements nature as the gender gap in culture appears to have its origins in pre-existing differences between men and women.

Alternative configurations. Up to now, we have only considered few emblematic cases among the large number of configurations that could occur. For instance, it could be the case that one trait dominates within one population, whereas a different one dominates within the population of the opposite sex[13], it may also be the case that one trait dominates within one population while cultural traits exhibit symmetric complementarity (or alternatively asymmetric complementarity) within the population of the opposite sex. In those configurations, the dynamical behavior of the economy is crucially shaped by the precise value of parameters A^s and B^s. Instead of providing an exhaustive analysis of each single case that could arise, let us focus on a specific configuration and discuss the new insights that we are able to gain from its analysis.

We assume that cultural traits exhibit symmetric complementarity within the population of males ($A^m > 0$ and $B^m > 0$) and asymmetric complementarity within the population of females ($A^f < 0$ and $B^f < 0$). The following proposition describes the long-run behavior of the society in that case and for relevant configurations of parameters.

Proposition 6. *Under Assumptions 1 and 2 and assuming that $A^m > 0$, $B^m > 0$, $A^f < 0$ and $B^f < 0$, we have:*

(i) *If $A^f \leq -A^m$ and $B^f \leq -B^m$, one unique interior globally stable steady-state co-exists with four unstable steady-states:* (0, 0), (1, 1), (0, 1) *and* (1, 0).
(ii) *If A^f is slightly higher than $-A^m$ and B^f is slightly higher than $-B^m$, three locally stable steady-states* (0, 0), (1, 1) *and one interior equilibrium co-exist with four unstable steady-states* (0, 1), (1, 0) *and two interior equilibria.*
(iii) *If A^f is widely higher than $-A^m$ and B^f is widely higher than $-B^m$, two locally stable steady-states* (0, 0) *and* (1, 1) *co-exist with three unstable steady-states* (0, 1), (1, 0) *and one interior equilibrium.*

To shed light on the point *i*, we first focus on the limit case where $A^m = -A^f$ and $B^m = -B^f$, such that $\rho^f = \rho^m = \rho$. When both q_t^f and q_t^m are lower than ρ, asymmetric complementarity favors the diffusion of the trait a for girls ($\Psi^f(q_t^m) > 0$), while symmetric complementarity favors the diffusion of the trait b for boys ($\Psi^m(q_t^f) < 0$). Consequently, both the ***mm*** and ***ff*** loci are below the 45° line. Yet, below this line we have $q_t^m < q_t^f$ such that, when considering altruistic motives only, incentives to transmit preference a to girls overcome incentives to transmit preference b to boys ($|\Psi^f(q_t^m)| > |\Psi^m(q_t^f)|$). Hence, the ***ff*** locus is located below the ***mm*** one. Symmetric reasoning applies when both q^f and q_t^m are higher than ρ (see Fig. 6(a)). Let us now describe how the distribution of cultural traits evolves overtime. As usual, cultural intolerance effect moves q_t^f and q_t^m in the area located between the two loci. There, altruistic motives dominate cultural intolerance for women while the reverse is true for men. Hence, for (q_t^f, q_t^m) below (resp. above) the (ρ, ρ) steady-state, both q_t^f and q_t^m increase (resp. decrease) and the interior steady-state is stable. Starting from $A^m = -A^f$ and $B^m = -B^f$, a fall in A^m or/and B^m would simply mitigate the strength of altruistic motives within the population m. Thus, the mechanisms previously sketched would be reinforced. Consequently, the interior steady state is still globally stable when $A^f \leq -A^m$ and $B^f \leq -B^m$ (see Fig. 6(b)).

In order to figure out item (ii) of Proposition 6, we can combine our analysis above (where $A^m = -A^f$ and $B^m = -B^f$) with the results of Lemma 1. As stated in Lemma 1, as $A^f > -A^m$ and $B^f > -B^m$, both the (0, 0) and the (1, 1) steady-states are locally stable. Nevertheless, as A^m is close to $-A^f$ and B^m is close to $-A^f$, it remains true that, when we go away from the egalitarian steady-states, the altruism-driven incentives to socialize girls are higher than the ones for boys. Then, the interior stead-state is locally stable too. This configuration is illustrated by the phase diagram in Fig. 6(c).

Finally, let us consider the point (iii) of Proposition 6 where A^m and B^m are sufficiently large. The relative incentives, driven by altruistic motives, that parents have to socialize their children are always higher for boys than for girls. Hence, when q_t^f is low (resp. large), the ***mm*** locus is located below (resp. above) the ***ff*** one. It follows that, in the area between the two loci, altruistic motives dominate cultural intolerance for men while the reverse is true for women. Therefore, when this area is below (resp. above) the interior steady-state, both q_t^f and q_t^m decrease (resp. increase) and the inegalitarian steady states (0, 0) and (1, 1) are locally stable (see Fig. 6(d)).

4. Application

In the previous section, we have proposed a general framework in which socialization choices, which drive the evolution of preferences, may be isolated from economic choices. We have also shown that the dynamical behavior of the economy could be deduced from the analysis of the kind of complementarity that prevails within the household (signs of A^s and B^s). Hence, our model constitutes a tractable framework that could be easily used to analyze under which conditions culturally driven gender inequality may persist or vanish over times. In this section we propose to apply this general model to particular cultural traits related to the prescribed role of women within the society. More precisely, we combine the approaches of Fernández et al. (2004) by assuming that some men are better off when their

13 Such a configuration will arise in the application we consider in the next section.

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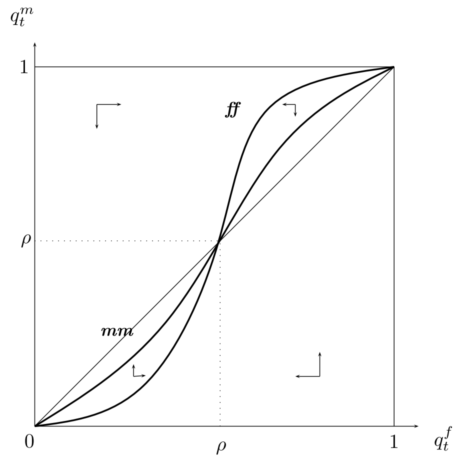
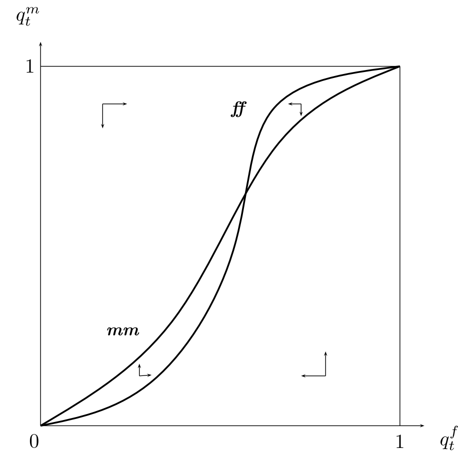
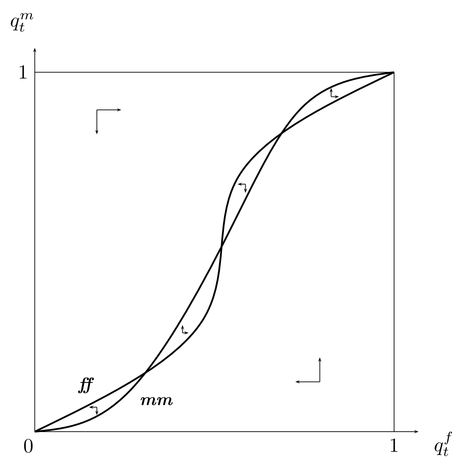


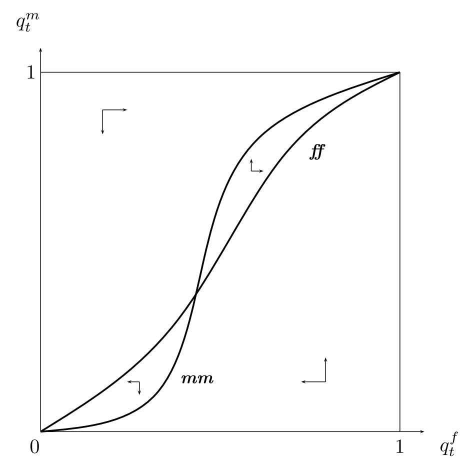
(a) $A^m = -A^f$ and $B^m = -B^f$.



(b) $A^m < -A^f$ and $B^m < -B^f$.



(c) A^f (B^f) slightly higher than $-A^m$ $(-B^m)$.



(d) A^f (B^f) widely higher than $-A^m$ $(-B^m)$.

Fig. 6. The phase diagram for $A^m > 0, B^m > 0$.

wife specializes in family activities, with the one of Escriche and co-authors (Escriche et al., 2004; Escriche, 2007) by considering some ''family oriented'' women get an extra utility from being involved in family responsibilities. The aim of this application is twofold. First, from a theoretical point of view, we show that the endogenization of the distribution of preferences within both populations enables to obtain some new results with respect to the existing literature linking gender inequalities to cultural evolutions. Second, our simple theoretical model is able to qualitatively reproduce some stylized facts concerning gender roles, FLFP and their evolutions. The presentation of our application is organized as follows. Section 4.1 exposes economic choices faced by individuals. Section 4.2 proposes an analysis of the dynamics of the economy. Finally, Section 4.3 discusses the relevance of the application from both a theoretical and an empirical perspective.

4.1. Economic decisions

Each individual is endowed with a fixed amount of time (or effort) that must be shared between the home and the labor market. To keep things simple, effort allocation choices are assumed to be dichotomous: Each spouse can either be *highly involved* in market activities (choosing a high effort $\bar{e}$) and do not spend any time in housework or *weakly involved* in market activities (choosing a low effort $\underline{e}$) and devote time to housework. The wage rate associated to a high involvement at the work place is denoted w, when choosing low effort an individual only receives $(1-\alpha)w$ with $\alpha \in (0, 1)$. Moreover we consider that some time or some money has to be spent in order to maintain the home and to take care of children. More precisely, one of the two spouses has to choose $\underline{e}$ in order to devote a minimum amount of time to housekeeping and child-rearing (the participation of the two spouses to housework is assumed to be useless) or, alternatively, a monetary amount c must be paid if both spouses decide to specialize in market activities. The cost c may be viewed as an institutional parameter (the development of child care facilities contributes to reduce c).

Each individual is assumed to derive utility from her/his own consumption that exactly equals the income she/he gets. In addition, individuals may differ in the definition they have of the prescribed role of women in the society: individuals of type a have modern views about this role; while individuals of type b hold more traditional views. More precisely, we denote μ_i the utility gain for a man of type i when his wife is highly involved in family activities and φ_i the utility gain for a woman of type i when participating to family activities, and we assume that:

$$\mu_i = \begin{cases} 0 & \text{if} \quad i = a \\ \mu > 0 & \text{if} \quad i = b \end{cases} \quad \text{and} \quad \varphi_i = \begin{cases} 0 & \text{if} \quad i = a \\ \varphi > 0 & \text{if} \quad i = b. \end{cases}$$

Put differently, individuals of type b aim at conforming to traditional role models according to which the man should be the breadwinner and the woman the homemaker. Moreover, we consider that a traditionalist woman suffers from an additional disutility $\delta > 0$ when her husband disagrees with her decision.

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This assumption is consistent with the fact that, according to the male breadwinner conception, the man should be the owner of the authority within the household. In fact, it essentially allows us to reduce the number of parametric restrictions that we have to deal with when analyzing the dynamics of the economy.[14]

The only economic decision that individuals have to made is a participation choice. Formally, an adult s, with the trait i and matched with a spouse of type j, has to choose a level of effort $e^{s,i}(j) \in \{\bar{e}, \underline{e}\}$. Within a family, effort choices are made in a non-cooperative way. These choices being given, the total income of the household is equally shared between the two spouses. The following payoff matrix summarizes the utility for a men of type i (in row) and a women of type j (in column) associated with the participation choices by spouses and abstracting from the disutility parameter δ:

	$\bar{e}$	$\underline{e}$
$\bar{e}$	$w - c/2$, $w - c/2$	$(1-\alpha/2)w + \mu_i$, $(1-\alpha/2)w + \varphi_j$
$\underline{e}$	$(1-\alpha/2)w$, $(1-\alpha/2)w$	$(1-\alpha)w + \mu_i$, $(1-\alpha)w + \varphi_j$

In order to reduce the number of equilibria, we focus on the case where Assumptions 3 and 4 hold:

Assumption 3. $c \in (\alpha w - 2\mu, \alpha w)$.

Assumption 4. $\delta > \max\{(\alpha w - c)/2 - \varphi, \varphi - (\alpha w - c)/2\}$.

The following lemma describes the Nash equilibrium of the game played by the two spouses:

Lemma 2. *Under Assumptions 3 and 4, the unique Nash Equilibrium of the game is:*

$$e^{s,i}(j) = \begin{cases} \underline{e} & \text{if } s = f \text{ and } (i,j) = (b,b) \\ \bar{e} & \text{otherwise.} \end{cases}$$

To figure out this result let us first underline that, under Assumption 3, the choice of $\bar{e}$ is a dominant strategy for men (whatever the cultural trait they hold) and for women of type a. In addition, a man of type a always prefers that his wife chooses $\bar{e}$ while a man of type b always prefers that his wife chooses $\underline{e}$. As a consequence, when matched with a spouse of type a (resp. b), a woman of type b would suffer from the disutility δ when choosing $\underline{e}$ (resp. $\bar{e}$). According to Assumption 4, this disutility parameter is high enough to ensure that a traditionalist woman always takes the decision that corresponds to her husband's will.[15]

From Lemma 2, we deduce the following expressions for the indirect utilities $u^{s,i}(j)$:

$$u^{m,i}(j) = \begin{cases} (1-\alpha/2)w + \mu & \text{if } (i,j) = (b,b) \\ w - c/2 & \text{otherwise} \end{cases} \quad (18)$$

$$u^{f,i}(j) = \begin{cases} (1-\alpha/2)w + \varphi & \text{if } (i,j) = (b,b) \\ w - c/2 & \text{otherwise} \end{cases} \quad (19)$$

implying that

$$A^m = 0, \qquad A^f = 0 \quad (20)$$

$$B^m = \mu - \frac{\alpha w - c}{2}, \qquad B^f = \varphi - \frac{\alpha w - c}{2}. \quad (21)$$

Let us notice that δ does not appear in the expressions of A^f and B^f as a woman of type b never takes a decision her husband could disagree with. Moreover, according to Assumption 3, $B^m > 0$ such that preference b dominates within the population of males. Finally, a decrease in c reduces the gain of both men and women to adopt preference b when they are matched with a person of type b such that the diffusion of child care facilities reduces the incentives that parents have to transmit traditional views about the role of women.

4.2. The dynamics

The following proposition describes how the distribution of cultural traits behaves in the long-run.

Proposition 7. *Under Assumptions 1–4 and considering $\mu > \varphi$, there is one threshold value $\bar{c}$ that belongs to the interval $[\alpha w - (\mu + \varphi), \alpha w - 2\varphi]$ such that:*

(1) *If $c \in [\bar{c}, \alpha w)$, $(0,0)$ is the unique globally stable steady-state while there are three unstable steady-states: $(1,1)$, $(0,1)$ and $(1,0)$.*
(2) *If $c \in (\alpha w - (\mu + \varphi), \bar{c})$, two locally stable steady-states $(0,0)$ and one interior equilibrium co-exist with four unstable steady-states $(1,1)$, $(0,1)$, $(1,0)$ and one unstable interior equilibrium.*
(3) *If $c \in (\alpha w - 2\mu, \alpha w - (\mu + \varphi)]$, $(1,1)$ is the unique globally stable steady-state while there are three unstable steady-states: $(0,0)$, $(0,1)$ and $(1,0)$.*

Let us first underline that when the availability of child-care facilities is very low ($c > \alpha w - 2\varphi$), $B^f > 0$ such that the trait b dominates within both populations and we know from Proposition 1 that $(0,0)$ is the unique stable steady-state.[16] Indeed, in that case, the monetary benefits associated to high involvement in market activities for both spouses are small compared to the utility benefits that traditionalist individuals derive when the wife takes care of the family. As a consequence, whatever the trait of the spouse she/he matched with, an individual is always weakly better off when being traditionalist.[17]

As c falls, the relative utility gains associated to the trait b recede (both B^f and B^m decrease) and as soon as $c < \alpha w - 2\varphi$, B^f becomes negative such that the trait a dominates within the population of women. In that configuration, the long-run situation depends on the intensity of altruistic incentives parents have to socialize their daughters, on the one hand, and their sons on the other one. As $A^m = A^f = 0$, this intensity is basically captured by $|\Psi^s(q_t^{-s})| = (1 - q_t^{-s})| - B^s|$ which is decreasing in q_t^{-s}. Indeed traditionalist individuals receive a utility gain (φ for females and μ for males) if and only if they are matched with a spouse who is traditionalist too. Moreover, as the trait b dominates within the population of males while the trait a dominates within the population of females, both the ***mm*** and the ***ff*** loci are below the 45° line where $q_t^m \leq q_t^f$. Hence, all other things being equal, socialization efforts driven by altruistic motives must be higher for girls than for boys as soon as (q_t^f, q_t^m) gets sufficiently close to these loci. Nonetheless, those socialization efforts also depend on the difference between B^m and $-B^f$ which is obviously decreasing in c. Having these mechanisms in mind, let us investigate more deeply the three cases listed in Proposition 7.

[14] This assumption implies that the husband's culture is an important determinant of his wife's work outcomes. This fact may be backed up by empirical findings of Fernández et al. (2004) or Fernández and Fogli (2009).

[15] Assumption 4 implies that $w - c/2 > (1-\alpha/2)w + \varphi - \delta$ such that $e^{f,b}(a) = \bar{e}$; and $w - c/2 - \delta < (1-\alpha/2)w + \varphi$ such that $e^{f,b}(b) = \underline{e}$.

[16] If the assumption $\mu > \varphi$ does not hold, Assumption 3 implies that c is always higher than $\alpha w - 2\varphi$ such that the joint distribution of cultural traits converges towards $(0,0)$ for all values of parameters compatible with Assumptions 3 and 4.

[17] In fact, when matched with a spouse of type a, an individual is fully indifferent between the two traits ($A^s = 0$) while, when matched with a spouse of type b, she/he is always better off when being b ($B^s > 0$).

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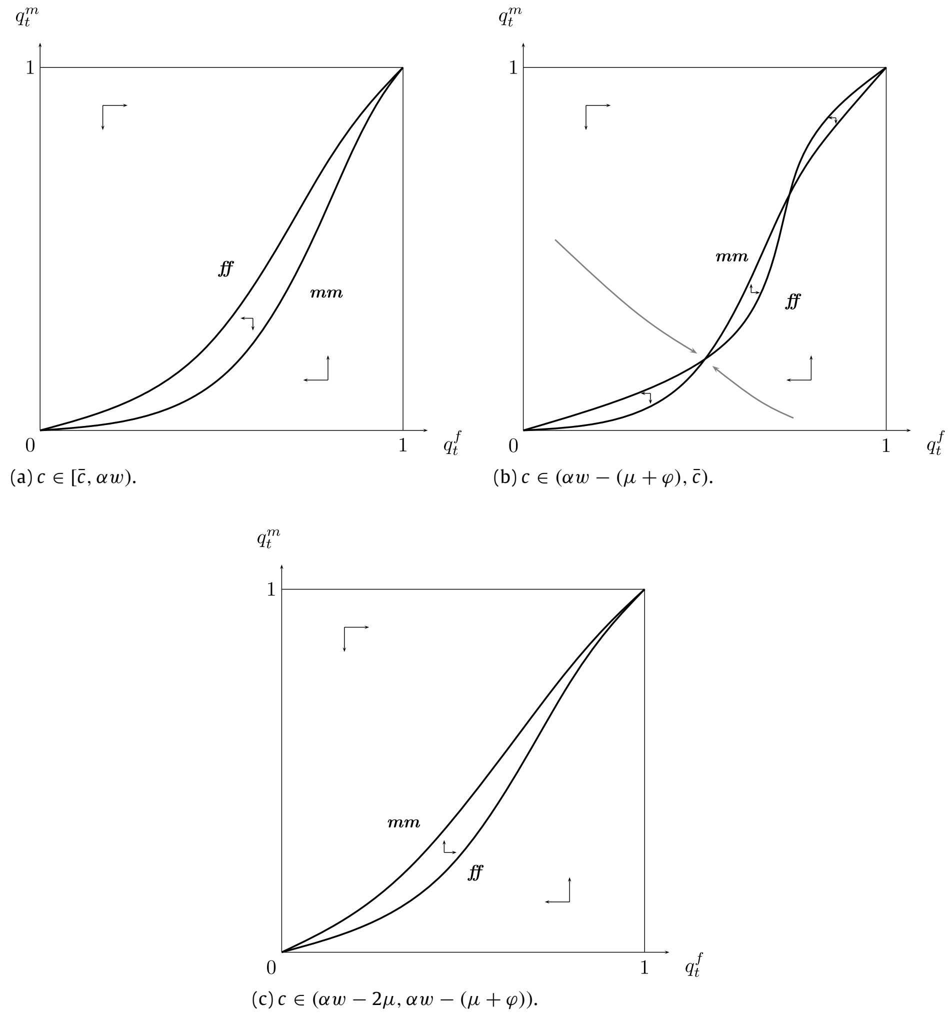

Fig. 7. The phase diagram.

When c is lower than $\alpha w - 2\varphi$ but still larger than $\bar{c}$, $-B^f$ remains low in comparison to B^m such that, the intensity of altruistic incentives to socialize children is higher for boys than for girls and the ***mm*** locus is always below the ***ff*** one. Accordingly, in the area between the two loci, altruistic motives dominate cultural intolerance for men while the reverse is true for women. Hence $(0, 0)$ is the unique stable steady-state (see the phase diagram in Fig. 7(a)).

A further reduction in c gets $-B^f$ and B^m even closer such that, for values of q_t^f slightly higher than q_t^m, the intensity of altruistic incentives to socialize girls overcomes the intensity of altruistic incentives to socialize boys. Graphically, the ***ff*** locus passes below the ***mm*** one for sufficiently balanced values of q_t^f (see the phase diagram in Fig. 7(b)). Hence, while the $(0, 0)$ steady-state is still locally stable, it co-exists with another interior steady-state which is locally stable too. As illustrated on the phase diagram there is also an interior unstable steady-state and the saddle paths that converge towards this steady-state share the state space within the basin of attraction of a *traditionalist equilibrium* $(0, 0)$ (below the saddle paths) and the basin of attraction of a *modern equilibrium* (above the saddle paths). The latter equilibrium is characterized by a larger proportion of type a individuals within both populations. Hence, at the *modern equilibrium* the proportion of (b, b) couples is much lower than at the *traditionalist* one such that the FLFP is significantly higher.

Finally, as soon as c becomes smaller than $\alpha w - (\mu + \varphi)$, we have $-B^m \geq B^f$ such that the condition $q_t^m \leq q_t^f$ becomes sufficient to ensure that $|\Psi^m(q_t^f)| \leq |\Psi^f(q_t^m)|$. Hence, the altruistic incentives to socialize girls always offset altruistic incentives to socialize boys implying that the ***mm*** locus is always above the ***ff*** one and $(1, 1)$ is the unique stable steady-state (see the phase diagram in Fig. 7(c)).

As previously discussed, for relatively balanced values of c (case (ii) in Proposition 7) multiple long-run equilibria arise. Focusing on that case, the following proposition allows us to analyze how a change in the value of the parameter c affects the likelihood for an economy to end up in the *traditionalist equilibrium* rather than the *modern* one.

Proposition 8. *Under Assumptions 1–4 and when $\mu > \varphi$ and $c \in (\alpha w - (\mu + \varphi), \bar{c})$, a rise in c shifts:*

- *the interior stable steady-state up to the right;*
- *the interior unstable steady-state down to the left.*

Proof. A rise in c increases both B^f and B^m. Then, the result of the proposition directly follows from Lemma 6. ■

According to Proposition 8, a fall in c enlarges (respectively reduces) the basin of attraction of the *modern* (resp. *traditionalist*) equilibrium. Indeed, by reducing the monetary costs associated to child care, a fall in c makes relatively less profitable the specialization of women in domestic activities. Accordingly, it reduces the utility gains for men and boosts the utility loss for women associated with trait b. Finally, it raises the likelihood that modern views about gender roles spread over time. Similarly, it increases the proportion of men and women of type a at the *modern equilibrium*.

4.3. Discussion

In this section, we provide a discussion of the results obtained in the above application along two main lines: (i) How our model contributes to the theoretical literature on the evolution of gender roles? (ii) What is the empirical relevance of our analysis?

From a theoretical point of view, the model presented in the application may be directly compared to the ones proposed by Fernández et al. (2004) and Escriche (2007). These two articles deal with the evolution of similar cultural traits but consider the distribution of traits as endogenous only within one population: in Fernández et al. (2004), men differ to the extent that they dislike to have a working wife while in Escriche (2007) women differ in the additional utility they get when taking care of the family. In those two papers, and differently from us, the long-run distribution of preference is unique and does not depend on initial conditions. Here, the multiplicity property directly stems from the existing interplays between cultural evolution within both populations. Hence, a full understanding of culturally driven gender inequality requires to take into account that preferences are endogenous for women as well as men.[18]

From an empirical perspective, the prediction of our application is twofold: (i) Multiple equilibria may arise meaning that two societies who are very close when considering their institutional and economic environments may converge towards quite different situation in terms of definition of gender roles and FLFP; (ii) a better access to child care facilities (fall in c) increases the likelihood for a society to end up in a situation characterized by egalitarian views about gender roles and high levels of FLFP. The prediction (i) may be interestingly related to a recent article by Steinhauer (2013) dealing with the labor market participation of women in Switzerland and focusing on the difference between the German-speaking and the French-speaking communities. More specifically, the author restricts his attention to cantons (states) that are cross-cut by the language border. He convincingly shows that, within those cantons, the two communities share the same institutional environment (in particular in terms of child-care availability) and that labor markets are well integrated. But, he finds that the labor force participation of mothers is significantly higher on the French-speaking compared to the German-speaking side, and interprets this result as a clear evidence of the role played by culture in participation choices of women. Steinhauer (2013) indeed reports that traditional views of gender roles, according to which women should restrict their participation to market activities in order to take care of the family, are much more prevalent among Swiss-Germans than Swiss-French. Our multiple equilibria story provides a relevant explanation for this pattern. Indeed, it implies that two societies[19] who share similar work-family policies and labor market opportunities (in the model, same values for the parameters c, w and α) may be characterized by quite different levels of FLFP only due to slight differences in the initial distribution of cultural traits.[20] Finally, let us underline that our model does not deny the role played by institutional and economic factors in the evolution of gender roles and FLFP. In particular, the result according to which a fall in c favors the participation of women to the labor market may be backed-up by several empirical studies. For instance Del Boca and Sauer (2009) show, at the cross-country level, that child care availability increases females employment rate (see also Lauer and Weber, 2003).

5. Conclusion

As stressed by recent empirical findings, many aspects of gender inequality may be explained by the existence of inegalitarian definitions of gender roles or gender differences in preferences. Our article aims at improving our understanding of those culturally driven gender inequalities by providing a tractable theoretical framework to model the dynamics of gender roles. To this end, we extend the seminal cultural transmission model proposed by Bisin and Verdier (2001) by considering that parents may choose to socialize their sons and their daughters differently. We are then able to characterize the joint dynamics of cultural traits within the population of men and women. Our framework may be used to analyze under which conditions culturally driven gender inequalities tend to persist or vanish over time. We provide a particular application focusing on the prescribed role of men and women within the society. We show that the evolution of the views about these roles within the populations of males and females may reinforce each others leading to the emergence of multiple equilibria. This result enables to figure out why the labor market outcomes of women may significantly differ between societies sharing similar economic and socio-institutional environments. We also show that technological or institutional changes, by transforming the way spouses interact within the household, may explain the evolution of gender roles. Our model could be extended in several directions. Relaxing the assumption of random matching on the marriage market in particular would generate some new motives for parents to socialize their children. Indeed, they would have incentives to transmit cultural traits which are likely to improve the attractiveness of their children on the marriage market. The analysis of these mechanisms may constitute an interesting path for further research.

Appendix A. Properties of the *ff* and *mm* loci

Lemma 3. *Under Assumption 1, both $Q^m(q_t^f)$ and $\chi^f(q_t^f)$ are increasing on $[0, 1]$ with $Q^m(0) = \chi^f(0) = 0$ and $Q^m(1) = \chi^f(1) = 1$.*

Proof. It directly comes from (17) that $Q^m(0) = 0$ and $Q^m(1) = 1$. The first derivative of $Q^m(q_t^f)$ is given in Box I From (11)–(13) we get that:

$$Q^{m\prime}(q_t^f) = \frac{\mathcal{N}^m(q_t^f)}{\left[q_t^f \Delta V_t^{m,a}(q_t^f) + (1-q_t^f)\Delta V_t^{m,b}(q_t^f)\right]^2} \tag{A.1}$$

with

$$\mathcal{N}^m(q_t^f) \equiv I^2 - [B^m - q_t^f(A^m + B^m)]^2 + 2q_t^f(1-q_t^f)(A^m + B^m)I.$$

[18] Let us underline that Escriche et al. (2004) also highlight the possibility of multiple long-run equilibria in a cultural transmission model where the distribution of preferences is endogenous only for women. However, they consider that firms do not offer the same jobs to men and women based on a kind of statistical discrimination. More specifically, the payoff that women get on the market depends on the distribution of preferences (in our model it would mean that $u^{f,i}(j)$ is a function of q_t^f). The multiplicity result they obtained is thus directly driven by this effect which is absent in our model.

[19] The German-speaking and French-speaking communities may be regarded as two distinct societies in the terms of the model as long as matching between men and women from different communities remains infrequent.

[20] Steinhauer (2013) proposes a model where women suffers a disutility (called identity cost) when working while being mother. The author argues that this model is also able to explain the labor participation gap between German-speaking and the French-speaking mothers. However, the result is driven by an exogenous differences in identity cost which is assumed to be higher for Swiss-Germans. Conversely, in our framework, the emergence of the gap is fully endogenous. In particular, it happens for identical values of the utility parameters μ and φ.

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$$Q^{m\prime}(q_t^f) = \frac{\Delta V_t^{m,a}(q_t^f)\Delta V_t^{m,b}(q_t^f) + q_t^f(1-q_t^f)\left[\Delta V_t^{m,a\prime}(q_t^f)\Delta V_t^{m,b}(q_t^f) - \Delta V_t^{m,a}(q_t^f)\Delta V_t^{m,b\prime}(q_t^f)\right]}{\left[q_t^f \Delta V_t^{m,a}(q_t^f) + (1-q_t^f)\Delta V_t^{m,b}(q_t^f)\right]^2}.$$

Box I.

As the denominator of (A.1) is positive, we have $Q^{m\prime}(q_t^f) \overset{\text{sign}}{=} \mathcal{N}(q_t^f)$. Moreover

$$\mathcal{N}^m(0) = I^2 - (B^m)^2 > 0 \quad \text{and} \quad \mathcal{N}^m(1) = I^2 - (A^m)^2 > 0$$

and $\mathcal{N}^m(q_t^f)$ is quadratic reaching an extremum for $q_t^f = (I + B^m)/(2I + A^m + B^m)$ with:

$$\mathcal{N}^m\left(\frac{I+B^m}{2I+A^m+B^m}\right) = \frac{2I(A^m+I)(B^m+I)}{A^m+B^m+2I} > 0.$$

Hence $\mathcal{N}^m(q_t^f)$ is positive as well as $Q^{m\prime}(q_t^f)$. Moreover, by definition of the inverse function:

$$Q^m(0) = 0 \Leftrightarrow \chi^f(0) = 0 \quad \text{and} \quad Q^m(1) = 1 \Leftrightarrow \chi^f(1) = 1$$

and

$$\chi^{f\prime}(q_t^f) = \frac{1}{Q^{f\prime}(\chi^f(q_t^f))}. \tag{A.2}$$

It directly comes from the properties of $Q^m(.)$ that $Q^f(.)$ is increasing on $[0, 1]$ such that $\chi^{f\prime}(q_t^f) > 0$. ■

Lemma 4. *Under Assumption 1:*

(i) $Q^{m\prime}(0) \lesseqqgtr 1$ *if* $B^m \gtreqqless 0$ *and* $Q^{m\prime}(1) \lesseqqgtr 1$ *if* $A^m \gtreqqless 0$.
(ii) $\chi^{f\prime}(0) \lesseqqgtr 1$ *if* $B^f \lesseqqgtr 0$ *and* $\chi^{f\prime}(1) \lesseqqgtr 1$ *if* $A^f \lesseqqgtr 0$.
(iii) $Q^{m\prime}(0) \lesseqqgtr \chi^{f\prime}(0)$ *if* $B^m \gtreqqless -B^f$ *and* $Q^{m\prime}(1) \lesseqqgtr \chi^{f\prime}(1)$ *if* $A^m \gtreqqless -A^f$.

Proof. We get from (A.1) that

$$Q^{m\prime}(0) = \frac{\Delta V_t^{m,a}(0)}{\Delta V_t^{m,b}(0)} = \frac{I - B^m}{I + B^m} \quad \text{and}$$

$$Q^{m\prime}(1) = \frac{\Delta V_t^{m,b}(1)}{\Delta V_t^{m,a}(1)} = \frac{I - A^m}{I + A^m}.$$

Moreover, when applying (A.2) we obtain:

$$\chi^{f\prime}(0) = \frac{1}{Q^{f\prime}(0)} = \frac{I + B^f}{I - B^f} \quad \text{and} \quad \chi^{f\prime}(1) = \frac{1}{Q^{f\prime}(1)} = \frac{I + A^f}{I - A^f}.$$

The results stated in the lemma directly follow. ■

Let us define $C^s \equiv A^s + B^s$ and the third degree polynomial

$$\begin{aligned}\mathcal{P}^s(q_t^{-s}) \equiv\ & C^s\left\{\Delta V_t^{s,b}(q_t^{-s}) - \Delta V_t^{s,a}(q_t^{-s}) + 2I(1-2q_t^{-s})\right\}\\ & \times\left[I + (1-2q_t^{-s})\right]\\ & - 2[\Delta V_t^{s,a}(q_t^{-s}) - \Delta V_t^{s,b}(q_t^{-s}) - (1-2q_t^{-s})C^s]\\ & \cdot [\Delta V_t^{s,a}(q_t^{-s})\Delta V_t^{s,b}(q_t^{-s}) + q_t^{-s}(1-q_t^{-s})C^s].\end{aligned}$$

Then we can claim that:

Lemma 5. *Under Assumption 1,* $\mathcal{P}^s(q_t^{-s})$ *is:*

- *Monotonously increasing on* $[0, 1]$ *if* $C^s < 0$.
- *Monotonously decreasing on* $[0, 1]$ *if* $C^s > 0$.

with

$$\mathcal{P}^s(0) = 4(B^s + I)[A^s I + B^s(2I - B^s)] \tag{A.3}$$

$$\mathcal{P}^s(1) = 4(A^s + I)[-B^s I - A^s(2I - A^s)]. \tag{A.4}$$

Moreover we have:

$$Q^{m\prime\prime}(q_t^f) \overset{\text{sign}}{=} \mathcal{P}^m(q_t^f) \quad \text{and} \quad \chi^{f\prime\prime}(q_t^f) \overset{\text{sign}}{=} \mathcal{P}^f(\chi^f(q_t^f)).$$

Proof. The first derivative of $\mathcal{P}^s(q_t^{-s})$ is

$$\begin{aligned}\mathcal{P}^{s\prime}(q_t^{-s}) =\ & 12C^s\{[A^s q_t^{-s} - B^s(1-q_t^{-s})]^2 - I^2\\ & - 2C^s q_t^{-s}(1-q_t^{-s})I\}\end{aligned}$$

such that

$$\mathcal{P}^{s\prime}(0) = 12C^s(B^s - I)(B^s + I) \overset{\text{sign}}{=} -C^s$$

$$\mathcal{P}^{s\prime}(1) = 12C^s(A^s - I)(A^s + I) \overset{\text{sign}}{=} -C^s$$

and

$$\begin{aligned}\mathcal{P}^{s\prime\prime}(q_t^{-s}) &= 12C^s[q_t^{-s}(A^s - I) + (1-q_t^{-s})(B^s - I)]\\ &\overset{\text{sign}}{=} -C^s.\end{aligned} \tag{A.5}$$

Moreover, simple computations yield to $Q^{s\prime\prime}(q_t^{-s}) = \mathcal{P}(q_t^{-s})/[I - (1-2q_t^{-s})\Psi^s(q_t^{-s})]^3$ and to expressions (A.3) and (A.4). Under Assumption 1, $I - (1-2q_t^{-s})\Psi^s(q_t^{-s})$ is always positive such that $Q^{m\prime\prime}(q_t^f) \overset{\text{sign}}{=} \mathcal{P}^m(q_t^f)$. Finally, the first derivative of (A.2) yields to:

$$\chi^{f\prime\prime}(q_t^f) = \frac{-Q^{f\prime\prime}(\chi^f(q_t^f))\chi^{f\prime}(q_t^f)}{[Q^{f\prime}(\chi^f(q_t^f))]^2} \tag{A.6}$$

and, as $\chi^{f\prime}(q_t^f) > 0$ (see Lemma 3), $\chi^{f\prime\prime}(q_t^f) \overset{\text{sign}}{=} -Q^{f\prime\prime}(\chi^f(q_t^f)) \overset{\text{sign}}{=} -\mathcal{P}^f(\chi^f(q_t^f))$. ■

Lemma 6 (*Comparative Statics*)**.** *Under Assumptions 1 and 2:*

- *When A^m (resp. B^m) increases, the **mm** locus is shifted upward (resp. downward).*
- *When A^f (resp. B^f) increases, the **ff** locus is shifted downward (resp. upward).*

Proof. By differentiating $Q^s(q_t^{-s})$ with respect to A^s and B^s, and after some algebra, we obtain:

$$\frac{dQ^s(q_t^{-s})}{dA^s} = \frac{2I(1-q_t^{-s})(q_t^{-s})^2}{[q_t^{-s}\Delta V_t^{s,a}(q_t^{-s}) + (1-q_t^{-s})\Delta V_t^{s,b}(q_t^{-s})]^2} > 0$$

$$\frac{dQ^s(q_t^{-s})}{dB^s} = \frac{-2I(1-q_t^{-s})^2 q_t^{-s}}{[q_t^{-s}\Delta V_t^{s,a}(q_t^{-s}) + (1-q_t^{-s})\Delta V_t^{s,b}(q_t^{-s})]^2} < 0.$$

Hence, by the properties of the inverse function, we must also have $\frac{d\chi^{-s}(q_t^{-s})}{dA^s} < 0$ and $\frac{d\chi^{-s}(q_t^{-s})}{dB^s} > 0$. ■

Lemma 7 (*Stability of the **mm** and **ff** Loci*)**.** *Under Assumptions 1 and 2:*

- *Considering q_t^f as given, the **mm** locus is globally stable.*
- *Considering q_t^m as given, the **ff** locus is globally stable.*

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Proof. From (9) and (10), for a given value of q_t^f, $\tau_t^{m,a}$ is decreasing in q_t^m while $\tau_t^{m,b}$ is increasing in q_t^m. Moreover, ***mm*** is defined as the locus where $q_t^f \tau_t^{m,a} = (1-q_t^f)\tau_t^{m,b}$. Hence, starting from this locus, a decrease (resp. an increase) in q_t^m implies $q_t^m \tau_t^{f,a} >$ (resp. $<$) $(1-q_t^m)\tau_t^{f,b}$ such that $q_{t+1}^m >$ (resp. $<$)q_t^m. A symmetric reasoning applies for the ***ff*** locus. ■

Appendix B. Proofs

Proof of Proposition 1. *Preference a dominates within both populations*. By Lemma 3, the ***mm*** and ***ff*** loci are monotonously increasing and pass through the points (0, 0) and (1, 1). By Lemma 4, the slope of the ***mm*** (resp. ***ff***) locus in (0, 0) is higher (resp. lower) than one while the slope of the ***mm*** (resp. ***ff***) locus in (1, 1) is lower (resp. higher) than one. By Lemma 5, the sign of both $Q^{m\prime\prime}(q_t^f)$ and $Q^{f\prime\prime}(q_t^f)$ changes at most once on [0, 1]. Putting these results together we deduce that, for $(q_t^f, q_t^m) \in [0,1]^2$, the ***mm*** locus is located below the ***ff*** ones and the two loci do not meet on (0, 1). The stability properties of the steady-states (0, 0) and (1, 1) are directly deduced from Lemma 7.

Preference b dominates within both populations. By Lemma 3, the ***mm*** and ***ff*** loci are monotonously increasing and pass through the points (0, 0) and (1, 1). By Lemma 4, the slope of the ***ff*** (resp. ***mm***) locus in (0, 0) is higher (resp. lower) than one while the slope of the ***ff*** (resp. ***mm***) locus in (1, 1) is lower (resp. higher) than one. By Lemma 5, the sign of both $Q^{m\prime\prime}(q_t^f)$ and $Q^{f\prime\prime}(q_t^f)$ changes at most once on [0, 1]. Putting these results together we deduce that, for $(q_t^f, q_t^m) \in [0,1]^2$, the ***mm*** locus is located above the ***ff*** ones and the two loci do not meet on (0, 1). The stability properties of the steady-states (0, 0) and (1, 1) are directly deduced from Lemma 7. ■

Proof of Proposition 2. By Lemma 3, the ***mm*** and ***ff*** loci are monotonously increasing and pass through the points (0, 0) and (1, 1). By Lemma 4, the slope of the ***mm*** (resp. ***ff***) locus is lower (resp. higher) than one both at the point (0, 0) and (1, 1). By Lemma 5, the sign of both $Q^{m\prime\prime}(q_t^f)$ and $Q^{f\prime\prime}(q_t^f)$ changes at most once on [0, 1] such that the ***mm*** (resp. ***ff***) locus must be first convex and then concave (resp. first concave and then convex). Putting these results together we deduce that the two loci cross in (0, 0), (1, 1) and at one point $(\bar{q}^f, \bar{q}^m) \in (0,1)^2$ and that the ***ff*** locus is located above (resp. below) the ***mm*** locus when $q_t^f < \bar{q}^f$ (resp. $q_t^f > \bar{q}^f$). The stability properties of the steady-states are directly deduced from Lemma 7. ■

Proof of Proposition 4. By Lemma 3, the ***mm*** and ***ff*** loci are monotonously increasing and pass through the points (0, 0) and (1, 1). By Lemma 4, the slope of the ***mm*** (resp. ***ff***) locus is higher (resp. lower) than one both at the point (0, 0) and (1, 1). By Lemma 5, the sign of both $Q^{m\prime\prime}(q_t^f)$ and $Q^{f\prime\prime}(q_t^f)$ changes at most once on [0, 1] such that the ***mm*** (resp. ***ff***) locus must be first concave and then convex (resp. first convex and then concave). Putting these results together we deduce that the two loci cross in (0, 0), (1, 1) and at one point $(\bar{q}^f, \bar{q}^m) \in (0,1)^2$ and that the ***ff*** locus is located below (resp. above) the ***mm*** locus when $q_t^f < \bar{q}^f$ (resp. $q_t^f > \bar{q}^f$). The stability properties of the steady-states are directly deduced from Lemma 7. ■

Proof of Proposition 5. The two first points of the proposition directly follow from comparative statics results formulated in Lemma 6. Concerning the other points, we know that the ***mm*** locus is first concave and then convex (with a slope higher than one at the points (0, 0) and (1, 1)) and the ***ff*** locus is first convex and then concave (with a slope lower than one at the points (0, 0) and (1, 1)) such that the two loci meet only once. Moreover, the ***mm*** (resp. ***ff***) locus crosses the 45° line when $q_t^f = \rho^m$ (resp. $q_t^m = \rho^f$). Then:

- If $\rho^m > \rho^f$: the ***ff*** locus crosses the 45° line before the ***mm*** one such that the point $(\bar{q}^f, \bar{q}^m)$ where the two loci meet must belong to the interval $(\rho^f, \rho^m)^2$ and be located above the first diagonal implying that $\bar{q}^f < \bar{q}^m$.
- If $\rho^m = \rho^f$: the two loci cross the first diagonal at the same point (ρ, ρ).
- If $\rho^m < \rho^f$: the ***mm*** locus crosses the 45° line before the ***ff*** one such that the point $(\bar{q}^f, \bar{q}^m)$ where the two loci meet must belong to the interval $(\rho^m, \rho^f)^2$ and be located below the first diagonal implying that $\bar{q}^f > \bar{q}^m$. ■

Proof of Proposition 6. Through expression (15), an interior equilibrium may be defined as a pair (q_t^f, q_t^m) such that the two following conditions are simultaneously satisfied:

$$q_t^f k(1-q_t^m)\Delta V^{m,a}(q_t^f) = (1-q_t^f)kq_t^m \Delta V^{m,b}(q_t^f) \tag{B.1}$$

$$q_t^m k(1-q_t^f)\Delta V^{f,a}(q_t^m) = (1-q_t^m)kq_t^f \Delta V^{f,b}(q_t^m). \tag{B.2}$$

Combining those two conditions, we obtain:

$$\frac{\Delta V^{m,a}(q_t^f)}{\Delta V^{f,b}(q_t^m)} = \frac{\Delta V^{m,b}(q_t^f)}{\Delta V^{f,a}(q_t^m)}. \tag{B.3}$$

Using expressions (11) and (12) and after some algebra, expression (B.3) yields to the following relationship between q_t^m and q_t^f that must be satisfied for each interior equilibrium:

$$q_t^m = \frac{B^m + B^f - q_t^f(A^m + B^m)}{A^f + B^f} \equiv \Theta^m(q_t^f). \tag{B.4}$$

Hence, interior steady-states are given by the crossing points between $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$. As cultural traits exhibit asymmetric complementarity within the population of males, we know that $Q^m(q_t^f)$ is increasing, first convex and then concave, with a slope in 0 and 1 lower than one. Then:

(i) If $A^f \leq -A^m$ and $B^f \leq -B^m$, $\Theta^m(0) \in [0,1)$ and $\Theta^m(1) \in (0,1]$ such that $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ cross only once on (0, 1). When $A^f = -A^m$ and $B^f = -B^m$, the crossing point between $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ is (ρ, ρ) with $\rho = \rho^m = \rho^f$. Using Eqs. (A.1) and (A.2), we obtain the following expressions for the slope of the ***mm*** and the ***ff*** loci at this point:

$$\left.\frac{\partial Q^m(q_t^f)}{\partial q_t^f}\right|_{q_t^f=\rho^m} = \frac{\mathcal{N}^m(\rho^m)}{\left[\rho^m \Delta V_t^{m,a}(\rho^m) + (1-\rho^m)\Delta V_t^{m,b}(\rho^m)\right]^2} = 1 + \frac{2A^m B^m}{A^m + B^m}$$

$$\left.\frac{\partial \chi^f(q_t^f)}{\partial q_t^f}\right|_{q_t^f=\rho^m} = \left.\frac{1}{Q^{f\prime}(\chi^f(q_t^f))}\right|_{q_t^f=\rho^m} = \left.\frac{1}{Q^{f\prime}(q_t^m)}\right|_{q_t^m=\rho^f} = \frac{1}{1+\frac{2A^f B^f}{A^f+B^f}} = \frac{1}{1-\frac{2A^m B^m}{A^m+B^m}}$$

such that we have

$$\left.\frac{\partial \chi^f(q_t^f)}{\partial q_t^f}\right|_{q_t^f=\rho^m} > \left.\frac{\partial Q^m(q_t^f)}{\partial q_t^f}\right|_{q_t^f=\rho^m}$$

meaning that the ***ff*** locus crosses the ***mm*** one upward in (ρ, ρ). The stability property of this interior steady state directly follows from Lemma 7. When $A^f < -A^m$ or $B^f < -B^m$, the stability properties of the steady-states are directly deduced from Lemmas 1 and 7.

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(ii) If A^f slightly higher than $-A^m$ and B^f slightly higher than $-B^m$, $\Theta^m(0)$ is slightly lower than 0 and $\Theta^m(1)$ slightly higher than 1 such that $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ cross third times on $(0, 1)$. The stability properties of the steady-states are directly deduced from Lemmas 1 and 7.

(iii) If A^f widely higher than $-A^m$ and B^f widely higher than $-B^m$, $\Theta^m(0)$ is widely lower than 0 and $\Theta^m(1)$ widely higher than 1 such that $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ cross only once on $(0, 1)$. The stability properties of the steady-states are directly deduced from Lemmas 1 and 7. ■

Proof of Proposition 7. The condition $c \geq \alpha w - 2\varphi$ implies that $B^f \geq 0$ such that $(0, 0)$ is the unique globally stable steady state (Proposition 1). Then we can restrict our attention to the configuration $c \in (\alpha w - 2\mu, \alpha w - 2\varphi)$ such that $B^m > 0$ and $B^f < 0$. Interior steady-states are given by the crossing points between $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ (see the proof of Proposition 6). We know from Lemma 5 that the sign of $Q^{m\prime\prime}(q_t^f) \overset{\text{sign}}{=} \mathcal{P}^m(q_t^f)$. By Lemma 5, we know that $\mathcal{P}^m(q_t^f)$ is monotonously decreasing in q_t^f. Moreover, as $A^m = 0$ and $\Delta > B^m$ (Assumption 1) we have:

$$\mathcal{P}^m(0) = 4(B^m + \Delta)[B^m(2\Delta - B^m)] > 0 \tag{B.5}$$

$$\mathcal{P}^m(1) = -4\Delta^2 B^m < 0. \tag{B.6}$$

Hence, the ***mm*** locus is increasing, first convex and then concave on $[0, 1]$ with a slope in 0 lower than one and a slope in 1 equal to one (Lemma 4). The slope of $\Theta^m(q_t^f)$ is $-B^m/B^f > 0$, $\Theta^m(0) = (B^f + B^m)/B^f$ and $\Theta^m(1) = 1$. Then when c is close to $\alpha w - 2\varphi$ (B^f close to zero), $\Theta^m(0)$ is very low such that $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ do not cross on $(0, 1)$. Conversely, when c is close (but smaller) to $\alpha w - (\mu + \varphi)$ (B^f slightly lower to $-B^m$), $\Theta^m(0)$ is lower but close to zero such that $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ cross twice on $(0, 1)$. Finally when $c \in (\alpha w - 2\mu, \alpha w - (\mu + \varphi))$, $\Theta^m(0)$ is positive such that $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ do not cross on $(0, 1)$. We deduce from this analysis that there is one threshold $\bar{c}$ that belongs to the interval $[\alpha w - (\mu + \varphi), \alpha w - 2\varphi]$ such that:

(i) If $c \in [\bar{c}, \alpha w - 2\varphi)$, $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ do not cross on $(0, 1)$. The stability properties of the steady-states are directly deduced from Lemmas 1 and 7.

(ii) If $c \in (\alpha w - (\mu + \varphi), \bar{c})$, $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ cross twice on $(0, 1)$. The stability properties of the steady-states are directly deduced from Lemmas 1 and 7.

(iii) If $c \in (\alpha w - 2\mu, \alpha w - (\mu + \varphi)]$, $\Theta^m(q_t^f)$ and $Q^m(q_t^f)$ do not cross on $(0, 1)$. The stability properties of the steady-states are directly deduced from Lemmas 1 and 7. ■

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