Advanced Macroeconomics

Lecture 1: Populations in economics

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Practical information

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Schedule of the course:

- Monday Feb. 12: 2:00-06:15 pm
- Tuesday Feb. 13: 4:00-06:00 pm
- Tuesday Feb. 20: 4:00-06:00 pm
- Monday March 05: 2:00-06:15 pm
- Tuesday March 06: 4:00-06:00 pm
- Monday March 26: 4:15-06:15 pm
- Monday Apr. 16: 10:45-12:45 am
- Monday Apr. 16: 16:15-18:15 pm
- Tuesday Apr. 16: 10:45-12:40 am

Outline of the course

Introductory Lecture

- Short video on the very long run dynamics of populations and prosperity
- General context
- The simplest economic model of fertility
- The simplest model of collective decisions on fertility
- Illustrative extensions to child mortality, longevity, endogenous mortality
- Optimality properties of the Beckerian model

Outline of the course

Lecture 1: Very long run determinants of economic growth and income disparities (7:30)

- Unified Growth Theory: some statements, refreshers and the new contributions (4:30)
  - The three phases of economic growth
  - The UGT: main theoretical framework
  - First refinements of the UGT
  - Pre-condition for the existence of UGT main mechanisms

Outline of the course

Lecture 1: Very long run determinants of economic growth and income disparities (7:30)

- Going beyond the UGT and analyzing the very long run determinants of growth (3:00)
    - Not a new idea: Guns, Germs and Steel
    - The funding contribution of Acemoglu on institutions (colonization) and the reversal of Fortunes
    - The global view of Galor on comparative development
    - Refinements of Galor's contributions
    - The alternative movement: origins of success and failures in the middle age

Outline of the course

Lecture 2: A series of hot topics in the field (7:30)

- Childlessness and higher order parities
- Economic crisis, economic booms and reproduction
- Wars, conflicts and fertility
- Culture, economy and population:
  - Becker and Stigler's critics
  - Measures of cultural transmission and preferences transmission
  - Theories of cultural transmission and preferences transmission
  - Applications

Students' Evaluation

1. MCQ on the class content: 20 questions with -0.25 in case of faulty answer, 0 in case of no answer and 1 in case of success. 50% of the final grade.
2. The philosophy of the course is to show that the long run dynamics of economies is determined, at least partially, by factors which are considered as non-economic. I will also show how these non-economic factors may be integrated in non-had-hoc ways into economic models. In a sense, this is true for any economic problem, be it a static or a dynamic. Each student will have to write a 2-3 pages statement about how their dissertation topic (MA Dissertation or PhD Dissertation) is dependent on the cultural, historical, geographical, social and/or demographic context they are taking place in and they will have to propose a way to take this into account in a non ad-hoc way. It will have to be presented in front of the class (30 minutes). 50% of the final grade.

Outline of the lecture

1. Why economists should worry about population dynamics?
2. Economic analysis of fertility behaviors: from Becker to Chiappori
3. Optimality properties of the Beckerian model

Why should economists worry about population dynamics

A brief and visual argument...

VIDEO

What getting from this?

- Maybe some Malthusian fears... it seems that we are on an exploding trend
- The settlement of humans on Earth is closely related to economic development
- Sustainability may be crucial

More arguments

Most of demographic phenomenon have an impact on economic decisions... the reverse being true:

- Longevity has an impact on health expenditures, education decisions (Ben-Porath), risky behaviors, retirement strategies, etc.
- Fertility has an impact on labor force supply of parents, public spendings, education decisions; it also shapes relations between generations, etc.
- The size of populations determine: depletion of exhaustible resources, geo-political equilibria, the degree of inequalities, labor supply, marginal productivities of labor and capital, etc.

These are only few arguments among others...

Economic analysis of fertility behaviors: from Becker to Chiappori

Economic analysis of fertility behaviors

Three models may be seen as the standard model of fertility:

- Imperfect altruism: Becker and Tomes (1976) and Becker and Lewis (1973)
- Dynastic altruism *à la* Barro and Becker (1988)
- Dynastic altruism *à la* Razin and Ben Zion (1975)

Despite their differences, they share their basic ingredients:

- Consumption, quantity of kids and quality of the kids enter the parental utility function
- Quality and quantity enter multiplicatively in the budget constraint: quality is not a pure public good inside the family
- Quality may be a source of positive externalities

Budget constraint

$$c + p_n(w,n)n + p_q(w,n)q = \omega + \Omega$$

Childrearing costs (pure quantity costs), in terms of goods and/or time:

- Time cost of raising children when $\frac{\partial p_n(w,n)}{\partial w} > 0$.
- Good cost if $p_n(0,n) > 0$
- Economies of scale in childrearing activities when $\frac{\partial p_n(w,n)}{\partial n} < 0$.
- Childrearing time is a pure public good for kids when $\frac{\partial p_n(w,n)}{\partial n} = 0$
- Childrearing time is a pure private good for kids when $\frac{\partial p_n(w,n)}{\partial n} = \bar{p}_n \in \mathbb{R}$

Budget constraint

$$c + p_n(w,n)n + p_q(w,n)q = \omega + \Omega$$

Costs attached to quality:

- Time cost of providing quality to children when $\frac{\partial p_q(w,n)}{\partial w} > 0$.
- Good cost if $p_q(0,n) > 0$
- Economies of scale in education activities when $\frac{\partial p_q(w,n)}{\partial n} < 0$.
- Education is a pure public good for kids when $\frac{\partial p_q(w,n)}{\partial n} = 0$
- Education is a pure private good for kids when $\frac{\partial p_q(w,n)}{\partial n} = p_q \in \mathbb{R}$

Remark: total education costs depend on n as soon as education is not a pure public good for kids.

Utility function and altruism

We have to introduce time subscripts:

1. Imperfect altruism: $V_t = u(c_t, n_t, q_{t+1})$
2. Dynastic altruism *à la* Barro and Becker: $V_t = u(c_t) + n_t\beta(n_t)V_{t+1}$ with $\beta'(n_t) < 0$
3. Dynastic altruism *à la* Razin and Ben Zion: $V_t = u(c_t, n_t) + \beta V_{t+1}$ with $\beta'(n_t) < 0$

In each configuration, $u(.)$ is increasing and concave wrt to each argument and respects Inada conditions

A widely used specification

UGT and other OLG models use a simple version with joy of giving type of altruism, time costs for childraising and quality as a pure private good for kids

Assumptions:

- Utility: $V_t = \alpha \ln c_t + \beta \ln n_t + \gamma \ln q_{t+1}$
- Budget: $c_t + \phi w_t n_t + \eta n_t q_{t+1} = w_t + \Omega_t$
- Technological constraint: $n_t \in [0, \frac{1}{\phi}]$ while $n_t = 0 \Rightarrow q_{t+1} = 0$
- Positivity constraints: $c_t \geq 0$, $n_t \geq 0$, $q_{t+1} \geq 0$
- $\beta > \gamma$

A widely used specification

Behaviors:

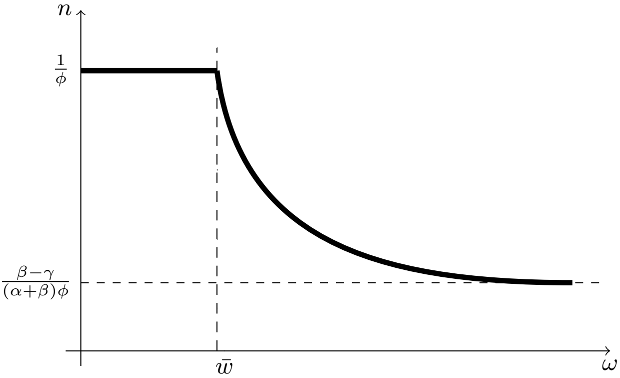
Let's denote $\bar{w} \equiv \frac{\beta-\gamma}{\alpha+\beta+\gamma}\Omega$

$$c^* = \begin{cases} \omega + \Omega & if \quad \omega \leq \bar{w} \\ \frac{\alpha}{\alpha+\beta}(\omega + \Omega) & if \quad \omega > \bar{w} \end{cases}$$

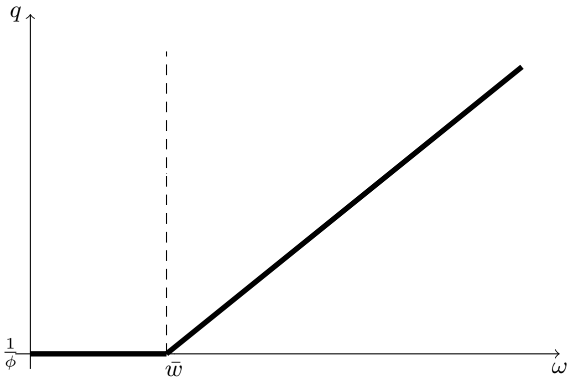
$$n^* = \begin{cases} \frac{1}{\phi} & if \quad \omega \leq \bar{w} \\ \frac{\beta-\gamma}{\alpha+\beta}\frac{\omega+\Omega}{\phi\omega} & if \quad \omega > \bar{w} \end{cases}$$

$$q^*_{t+1} = \begin{cases} 0 & if \quad \omega \leq \bar{w} \\ \frac{\gamma}{\beta-\gamma}\frac{\phi}{\eta}\omega & if \quad \omega > \bar{w} \end{cases}$$

A widely used specification



A widely used specification



A widely used specification

Comparative statics:

- Higher wages may imply a substitution of quality to quantity for people who are rich enough, quality being mainly human capital, financial bequest, health, etc.
- An increase in non labor income Ω implies more quantity and less quality
- Any exogenous process making ω increasing from one generation to another is able to provoke a fertility transition and a take-off of human capital average levels.

Obviously, many aspects are forgotten here:

- Dynastic altruism may be important
- Reproduction is not asexual
- Marital status may be important
- Not having kids is a possibility
- Quality and quantity may not be substitute but complements (see Jones and Schoonbroodt, 2009)
- ...

Simple extensions: child mortality

Let's assume that only a fraction μ_t of the new born survive to age 1 (infant mortality) or age 5 (child mortality). Let's also assume that parent only educate surviving kids and face time costs for raising kids such that:

- Utility: $V_t = \alpha \ln c_t + \beta \ln(\mu_t n_t) + \gamma \ln q_{t+1}$
- Budget: $c_t + (\xi + \zeta\mu_t)w_t n_t + \eta\mu_t n_t q_{t+1} = w_t + \Omega_t$
- Technological constraint: $n_t \in [0, \frac{1}{\xi+\zeta}]$ while $n_t = 0 \Rightarrow q_{t+1} = 0$
- Positivity constraints: $c_t \geq 0$, $n_t \geq 0$, $q_{t+1} \geq 0$
- $\beta > \gamma$

Straightforward resolution...

Simple extensions: others

- On longevity: de la Croix (2010)
- On inequalities: de la Croix and Doepke (2003), Galor and Zeira (2010)
- Property rights and efficiency: Tertilt and Schoonbroodt (2014)
- Endogenous labor supply and fertility: Blackburn and Cipriani (2002)
- Child mortality and uncertainty: Kalemli-Ozcan (2002)

Negotiation models of fertility

Fertility decisions may not be made by only one single person but by 2 persons within a couple...

Two major ways to model it:

- Collective non-cooperative decision process
- Collective cooperative decision process

... and many combinations of them.

We will focus on the cooperative process, see for instance Doepke and Kinderman (2016) for a non cooperative model and Gobbi (2017) for hybrid alternatives $\rightarrow$ No marriage and divorce for simplicity

Fertility models à la Chiappori

- Chiappori (1992) has developed models where households take their decisions in a collective and cooperative way. These models are not specific to fertility but may be applied to them.
- In this setting, couples maximize a common objective, let's denote it $W(x_f, x_m, g)$ where x_f denotes the vector of private consumption made by partner f, x_m that of partner m while g is a vector of public goods. We limit the presentation to the simplest version of the model, for a complete discussion see Chiappori and Doni (2011).
- In the simplest case, budget constraint is unique: pooled income and pooled spending:

$$p_f c_f + p_m c_m + p_g g = \omega_f + \omega_m + \Omega \tag{1}$$

Sets c_f and c_m may contain common goods either at the same price or not (popular cases of discriminations between men and women). The set of prices may depend on labor and non-labor income.

Fertility models à la Chiappori

- In its simplest version, the objective function of the couple is a weighted sum of individual utilities (no altruism):

$$W(x_f, x_m, g) = \theta(\zeta^f, \zeta^m)V^f(x_f, g) + (1 - \theta(\zeta^f, \zeta^m))V^m(x_m, g) \quad (2)$$

  We may have gender differences in preferences while the negotiation power of each partner depends on his/her set of personal characteristics ζ^s while the shape of the function $\theta(.,.)$ may depend on the institutional and cultural context

- Let's assume a separable utility case such that:

$$V^f(x_f, g) = u^f(x_f) + \mathcal{D}^f(g) \text{ and } V^m(x_m, g) = u^m(x_m) + \mathcal{D}^m(g)$$

Fertility models à la Chiappori

Let's assume that the vector of public goods is composed of children in quantity n and of quality h such that:

$$\mathcal{D}^s(g) = d_n^s(n) + d_h^s(h)$$

We then get that a couple $\{m, f\}$ will maximize its objective function 2 subject to the following budget constraint:

$$c^f + c^m + \phi(\alpha w^f + (1-\alpha)w^m)n + \rho nh = w^f + w^m + \Omega^f + \Omega^m \tag{3}$$

Fertility models à la Chiappori: logarithmic case

We make the assumption that functions u and d are logarithmic and $\zeta^s = w^s$ such that:

$$W(x_f, x_m, n, h) = \theta(w^f, w^m)\left[\alpha \ln x^f + \beta^f \ln n + \gamma^f \ln h\right] \tag{4}$$

$$+(1-\theta(w^f, w^m))\left[\alpha \ln x^m + \beta^m \ln n + \gamma^m \ln h\right] \tag{5}$$

- No gender differences in preferences for consumption to alleviate notations
- Gender differences in preferences for kids: see Baudin and Hiller (2017) for a justification
- $\frac{\partial\theta(w^f,w^m)}{\partial w^f} > 0$ while $\frac{\partial\theta(w^f,w^m)}{\partial w^m} < 0$ and $0 < \theta(0, w^m) < \theta(w^f, 0) < 1$

Fertility models à la Chiappori: logarithmic case

Budget constraint:

$$c^f + c^m + \phi\left(\lambda w^f + (1-\lambda) w^m\right) n + \rho n h = w^f + w^m + \Omega^f + \Omega^m \qquad (6)$$

where $(\phi, \lambda) \in [0,1]^2$, $(\Omega^f, \Omega^m, \rho) \in \mathbb{R}^{+3}$

Technological constraints:

- $(c^f, c^m, h) \in \mathbb{R}^{+3}$
- $n \in \left]0, \min\left(\frac{1}{\lambda\phi}, \frac{1}{(1-\lambda)\phi}\right)\right[$

Fertility models à la Chiappori: logarithmic case

Optimal decisions of a couple maximize (5) subject to (6) and technological constraints. In the interior regime, we get that:

$$
\begin{aligned}
x^{f^*} &= \frac{\theta\alpha(w^f + w^m + \Omega^f + \Omega^m)}{\alpha + \theta(\beta^f + \gamma^f) + (1-\theta)(\beta^m + \gamma^m)} \\
x^{m^*} &= \frac{(1-\theta)\alpha(w^f + w^m + \Omega^f + \Omega^m)}{\alpha + \theta(\beta^f + \gamma^f) + (1-\theta)(\beta^m + \gamma^m)} \\
n^* &= \frac{\theta\beta^f + (1-\theta)\beta^m}{\alpha + \theta(\beta^f + \gamma^f) + (1-\theta)(\beta^m + \gamma^m)} \cdot \frac{w^f + w^m + \Omega^f + \Omega^m}{\phi(\lambda w^f + (1-\lambda)w^m)} \\
h^* &= \frac{\theta\gamma^f + (1-\theta)\gamma^m}{\theta\beta^f + (1-\theta)\beta^m} \frac{\phi(\lambda w^f + (1-\lambda)w^m)}{\rho}
\end{aligned}
$$

Need a White-Board Explanation?

Fertility models à la Chiappori: logarithmic case

Comparative statics:

Total spending in kids represent a constant share of potential parental income (due to the log specification):

$$\frac{\phi(\lambda w^f + (1-\lambda) w^m) n^* + \rho h^*}{w^f + w^m + \Omega^f + \Omega^m} = \frac{\alpha}{\alpha + \theta(\beta^f + \gamma^f) + (1-\theta)(\beta^m + \gamma^m)}$$

The quality-quantity trade-off regarding wages becomes more complex:

- Usual effects: substitution-income effects
- Empowerment effects: relative weights of quantity and quality vary with wages

Fertility models à la Chiappori: logarithmic case

Comparative statics:

The quality-quantity trade-off regarding wages becomes more complex: let's have a look at quality only

$$h^* = f(w^f, w^m)\frac{\phi[\lambda w^f + (1-\lambda)w^m]}{\rho} \tag{7}$$

with:

$$f(w^f, w^m) = \frac{\theta(w^f, w^m)\gamma^f + (1-\theta(w^f, w^m))\gamma^m}{\theta(w^f, w^m)\beta^f + (1-\theta(w^f, w^m))\beta^m}$$

Fertility models à la Chiappori: logarithmic case

Comparative statics:

Differentiating equation (7) wrt w^f, we get that:

$$\frac{dh^*}{dw^f} = \frac{\partial f(w^f, w^m)}{\partial w^f} \cdot \frac{\phi[\lambda w^f + (1-\lambda)w^m]}{\rho} + \frac{\phi\lambda}{\rho} f(w^f, w^m)$$

A rapid inspection of the equation indicates that:

$\frac{\partial f(w^f, w^m)}{\partial w^f}$	$\beta^f > \beta^m$	$\beta^f \leq \beta^m$
$\gamma^f > \gamma^m$	?	≥ 0
$\gamma^f \leq \gamma^m$	≤ 0	?

Fertility models à la Chiappori: logarithmic case

Comparative statics:

We can show that:

$$\text{if } \beta^f > \beta^m \text{and } \gamma^f > \gamma^m \text{then } \frac{\partial f(w^f, w^m)}{\partial w^f} > 0$$

$$\iff \quad \frac{\gamma^f - \gamma^m}{\beta^f - \beta^m} > \frac{\theta\gamma^f + (1-\theta)\gamma^m}{\theta\beta^f + (1-\theta)\beta^m}$$

$$\text{if } \beta^f < \beta^m \text{and } \gamma^f < \gamma^m \text{then } \frac{\partial f(w^f, w^m)}{\partial w^f} > 0$$

$$\iff \quad \frac{\gamma^f - \gamma^m}{\beta^f - \beta^m} < \frac{\theta\gamma^f + (1-\theta)\gamma^m}{\theta\beta^f + (1-\theta)\beta^m}$$

The substitution of quality to quantity may be ruled out when the distance between the woman's relative aspirations for the quantity of kids are not too strongly superior to that of the man.

Fertility models à la Chiappori: a conclusion

One can think to many extensions of this framework regarding utility functions, what determine θ (see Chiappori and Doni, 2011) and endogenous marriage and divorce decisions

We will see one of them in some weeks

But in the end, the main idea will remain valid: the impact of an economic change will transit through the pure economic effects and empowerment effects.

Optimality properties of the Malthusian model

Optimality properties

The simplest model of endogenous fertility delivers very nice results, but... its optimality properties are more surprising

In the Standard Literature: general consensus about the legitimacy of policies subsidizing education. This legitimacy lies in the fact that education is a source of positive externalities (Hanushek & Welch, 2006)

Optimality properties

What does the standard model of fertility say?

- Because the size of generations is endogenous, the social discount rate can be different from the private one: endogenous discount rate. Two famous cases: Millian case vs Benthamite case $\rightarrow$ Samuelson Vs Deardorff
- Despite education is a source of positive externalities, it exists some cases where it is not optimal to subside education investments. On the contrary, taxing education spending can be optimal.
- When there only exist positive externalities on education choices and a Millian SWF, education subsidies have to be combined with, at least, another policy: a family policy that consists in taxing births.

These results crucially comes from the non linearity of the budget constraint in standard models of fertility and from the chosen social welfare function.

Optimality properties

OLG economy with L_t adult agents living for 2 periods and building perfect foresights:

- childhood: no consumption, only receive education from their parents
- adulthood: choose their consumption c_t, their number of children X_t and the education they provide to their children e_t

Families are monoparental for simplicity. Dynastic utility function *à la* Razin and Ben Zion [1975]:

$$V_t = \max \{u(C_t, X_t) + \beta V_{t+1}\} \tag{8}$$

Optimality properties

The Human Capital Production function:

Decreasing return to scale accumulation process, in line with De la Croix & Doepke [2003]):

$$h_{t+1} = l\left(e_t, h_t, \overline{h_t}\right),\ l_1' > 0, l_{11}'' \leq 0, l_2' > 0, l_{22}'' \leq 0, l_3' > 0, l_{33}'' \leq 0 \tag{9}$$

- There is an intrafamily transmission of human capital: the human capital of parents h_t positively influences the future human capital of children.
- The average level of human capital in the population $\bar{h}_t$ has a positive impact on the children's future human capital. This is an externality.

Let's notice that the function l is strictly increasing and concave regarding educational investment such that e_t can be expressed as a function of $\bar{h}_t$, h_t and h_{t+1} such that:

$$e_t = e\left(h_{t+1}, h_t, \bar{h}_t\right) \quad \text{where} \quad e_1' > 0, e_2' < 0, e_3' < 0. \tag{10}$$

Optimality properties

Parents faces the following non linear budget constraint:

$$C_t + \left[\frac{\sigma}{\xi} + \phi\right] w_t h_t X_t + \theta w_t h_t \Omega\left(X_t\right) \cdot e_t = w_t h_t \tag{11}$$

where $X_t \equiv \xi N_t$ denotes the number of surviving children at the end of period t and $\xi \in]0,1[$ the fraction of children who survive to age five.

Opportunity (time) cost to raise one child: $\phi w_t h_t$

Total cost of education is concave in X_t , one unit of education can benefit more than one child. Cost of giving e_t units of education to X_t children is $\theta w_t h_t \Omega\left(X_t\right) \cdot e_t$ with $\Omega^{'}\left(X_t\right) > 0$ and $\Omega^{''}\left(X_t\right) < 0$.

Notice that the non linearity of the budget constraint implies that the marginal cost of quantity depends on educational investment while, in turn, the marginal cost of quality (education) also depends on quantity.

Optimality properties

The Final Good: its price is normalized to one. Produced with a linear technology:

$$Y_t = AH_t \tag{12}$$

A is a productivity factor and H_t the total amount of human capital in the workforce.

At the labor market's equilibrium:

$$H_t = \left[1 - \left(\frac{\sigma}{\xi} + \phi\right) X_t - \theta e_t \Omega\left(X_t\right)\right] h_t L_t \tag{13}$$

As the labor market is competitive, the wage equals the workers' marginal productivity:

$$w_t = A \tag{14}$$

Optimality properties

Individual Behaviors

Iterating equation (8) from $t-1$ to $+\infty$, we get that a parent born in $t-1$ determines his/her optimal demands C_t^*, X_t^*, e_t^* by solving the following problem subject to (3):

$$\max_{C_s, X_s, e_s} V_t = \sum_{s=t}^{+\infty} \beta^{s-t} u\left(C_s, X_s\right) \tag{15}$$

Necessary condition: $\lim_{T\rightarrow+\infty} \beta^T u\left(C_T, X_T\right) = 0$. X_t being the number of surviving children, it is bounded by $\frac{1}{\frac{\sigma}{\xi}+\phi}$ the maximal number of children a wife can give birth to.

Optimality properties

Substituting C_t thanks to the budget constraint and using the definitions $e_t = e(h_{t+1}, h_t, \bar{h}_t)$ and $X_t = \frac{L_{t+1}}{L_t}$, this problem can be solved by maximizing the following objective with respect to L_{s+1} and h_{s+1} :

$$\max_{L_{s+1}, h_{s+1}} V_t =$$

$$\sum_{s=t}^{+\infty} \beta^{s-t} u\left(w_s h_s - \left[\frac{\sigma}{\xi} + \phi\right] w_s h_s \frac{L_{s+1}}{L_s} - \theta w_s h_s \Omega\left(\frac{L_{s+1}}{L_s}\right) \cdot e_s, \frac{L_{s+1}}{L_s}\right)$$

Optimality properties

First order conditions wrt to L_{s+1} and h_{s+1}:

$$-\beta \frac{X^*_{t+1}}{X^*_t} = \frac{u'_{X_t} - Ah^*_t \left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_t} e\left[h^*_{t+1}, \cdot, \cdot\right]\right) u'_{C_t}}{Ah^*_{t+1}\left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_{t+1}} e\left[h^*_{t+2}, \cdot, \cdot\right]\right) u'_{C_{t+1}} - u'_{X_{t+1}}} \quad (16)$$

$$\frac{u'_{C_{t+1}}}{u'_{C_t}} = \frac{\theta h^*_t \Omega\left(X^*_t\right) e'_1\left[h^*_{t+1}, \cdot, \cdot\right]}{\beta\left[1 - \left(\frac{\sigma}{\xi} + \phi\right) X^*_{t+1} - \theta \Omega\left(X^*_{t+1}\right)\left(e\left[h^*_{t+2}, \cdot, \cdot\right] + h^*_{t+1} e'_2\left[h^*_{t+2}, \cdot, \cdot\right]\right)\right]} \quad (17)$$

Optimality properties

The competitive equilibrium:

Defined by the sequence $\{C_t, X_t, h_t, \bar{h}_t, e_t, H_t, Y_t, w_t\}_{t=0}^{t=+\infty}$ satisfying the set of conditions $\{10, 11, 12, 13, , 14, 16, 17\}_{t=0}^{t=+\infty}$.

Some intuitions

- The competitive equilibrium cannot be optimal because of the existence of an externality on education choices: parents tend to underinvest in education.
- Intuitively, a distortive subsidy on education combined with a lump sum tax should correct this externality.
- However, as mentionned previously, this will not be true...

Optimality properties

The First Best Optimum

In order to analyze optimality in this model, two concepts have to be discussed: efficiency and the Social Welfare Function (SWF)

Efficiency when Fertility is endogenous

- Golosov et al [2007] define the concepts of $\mathcal{A}$ and $\mathcal{P}$ efficiency
- Conde Ruiz [2008] provides a very closed concept of $\mathcal{U}$-Efficiency
- Michel & Wigniolle [2007] define RC and CRC-Efficiency
- etc.

I will discuss my results in the light of $\mathcal{A}$-efficiency.

Optimality properties

Social Welfare Function: the choice of the Welfare function is not trivial because the size of the population is endogenous (no ethical problems à la Blackorby et al [2005,2006], no Rawlsian objectives à la Spiegel [1993].) I propose the following Social Welfare function

$$W_0 = \sum_{t=0}^{+\infty} \beta^t f\left(L_t\right) u\left(C_t, X_t\right) \tag{18}$$

where $f'\left(L_t\right) \geq 0$ and $u(C_t, X_t) > 0 \; \forall (C_t, X_t) > (0, 0)$.

$f(L_t)$ denotes the social preference for the population stock. Because $f'\left(L_t\right)$ is strictly positive, for a fixed per capita utility level $\overline{u}$, the Social Planner prefers larger populations.

Optimality properties

For tractability of results, I assume that $\frac{f(\iota)}{f(\eta)} = F\left(\frac{\iota}{\eta}\right)$ with $F'\left(\frac{\iota}{\eta}\right) > 0$, $F(1) = 1$ and $F(\cdot)$ being twice differentiable.

Following Jones & Schoonbroodt [2007,2009], the case where $u(C_t, X_t) < 0$ $\forall (C_t, X_t) > (0, 0)$ and $f'(L_t) < 0$ also makes sense and has to be fully analyzed. This is done in Baudin (2010) appendix B.

Optimality properties

Polar SWFs:

- Benthamite Social Preferences: $f(L_t) = L_t$
- Millian Social Planner: $f(L_t) = 1$

Non Millian social planner are characterized by something like an endogenous social discount rate $\beta f(L_t)$

Assumption: $\lim_{t \to +\infty} \beta^t F(\frac{1}{\phi^t}) = 0$

Under assumption 1, the social planner's objective is bounded. Example: if $f(L_t) = L_t^a$, then assumption 1 becomes $\phi > \beta^{\frac{1}{a}}$

Optimality properties

First best Optimum: At each date t, the resource constraint is:

$$C_t L_t = A h_t \left(1 - \left[\frac{\sigma}{\xi} + \phi\right] X_t - \theta \Omega\left(X_t\right) \cdot e_t\right) L_t \tag{19}$$

Then, the Social Planner has to maximize (18) with respect to $\{C_t, X_t, h_{t+1}\}_{t=0}^{t=+\infty}$ and subject to $\{(9), (19)\}_{t=0}^{t=+\infty}$.

Optimality properties

The social optimum is defined by the set $\left\{\widehat{C}_t, \widehat{X}_t, \widehat{e}_t, \widehat{h}_t, \widehat{h}_{t+1}, \quad \widehat{H}_t, \widehat{Y}_t\right\}_{t=0}^{t=+\infty}$ satisfying equations $\{(9), (12), (13), (19)\}_{t=0}^{t=+\infty}$ and, at each date t, the following FOC with respect to L_{t+1} and h_{t+1}:

$$-\frac{1}{\beta} = \frac{\varepsilon_L^f u(\cdot, \widehat{X}_{t+1}) + F\left(\widehat{X}_t\right) \frac{\widehat{X}_{t+1}}{\widehat{X}_t} \left[A\widehat{h}_{t+1}\left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_{t+1}} e\left[\widehat{h}_{t+2}, \cdot, \cdot\right]\right) u'_{C_{t+1}} - u'_{X_{t+1}}\right]}{u'_{X_t} - Ah_t\left(\frac{\sigma}{\xi} + \phi + \theta \Omega'_{X_t} e\left[\widehat{h}_{t+1}, \cdot, \cdot\right]\right) u'_{C_t}} \tag{20}$$

$$\frac{u'_{C_{t+1}}}{u'_{C_t}} = \frac{\theta h_t \Omega(X_t) e'_1\left[\widehat{h}_{t+1}, \cdot, \cdot\right]}{\beta F\left(\widehat{X}_t\right)\left[1 - \left(\frac{\sigma}{\xi} + \phi\right) \widehat{X}_{t+1} - \theta \Omega\left(\widehat{X}_{t+1}\right)\left(e\left[\widehat{h}_{t+2}, \cdot, \cdot\right] + \widehat{h}_{t+1}\left(e'_2\left[\widehat{h}_{t+2}, \cdot, \cdot\right] + e'_3\left[\widehat{h}_{t+2}, \cdot, \cdot\right]\right)\right)\right]} \tag{21}$$

where $\varepsilon_v^g \equiv \frac{\partial g}{\partial v} \frac{v}{g}$ denotes the elasticity of g with respect to v.

Optimality properties

At the social optimum, all of the existing externalities are taken into account:

1. a Lucas-type externality in the accumulation of human capital
2. when $f'(L_t)$ is strictly positive, parental preferences differ from the preferences of the Social Planner since parents do not value the population stock.

The government has to implement a public policy that makes the competitive equilibrium $\{C_t^*, X_t^*, e_t^*, h_t^*, h_{t+1}^*, H_t^*, Y_t^*\}_{t=0}^{t=+\infty}$ coincide with the social optimum $\left\{\widehat{C}_t, \widehat{X}_t, \widehat{e}_t, \widehat{h}_t, \widehat{h}_{t+1}, \widehat{H}_t, \widehat{Y}_t\right\}_{t=0}^{t=+\infty}$.

Optimality properties

Government budget constraint:

$$T_t = \lambda_t \theta e\left(h^*_{t+1}, h^*_t, \overline{h}^*_t\right) \Omega\left(X^*_t\right) A h^*_t - \frac{\Lambda_t}{\xi} X^*_t A h^*_t \tag{22}$$

Individual budget constraint:

$$C_t + \left[\frac{\sigma + \Lambda_t}{\xi} + \phi\right] w_t h_t X_t + \left(1 - \lambda_t\right) \theta w_t h_t \Omega\left(X_t\right) \cdot e_t = w_t h_t + T_t \tag{23}$$

Optimality properties

The competitive equilibrium is now defined by the set $\{C_t^*, X_t^*, e_t^*, h_t^*, h_{t+1}^*, H_t^*, Y_t^*, w_t^*\}_{t=0}^{t=+\infty}$ satisfying equations $\{(9), (12), (13), (14), (23)\}_{t=0}^{t=+\infty}$ and the following first order conditions with respect to L_{t+1} and h_{t+1} :

$$-\beta \frac{X_{t+1}}{X_t} = \frac{u'_{X_t} - Ah_t\left(\frac{\sigma+\Lambda_t}{\xi} + \phi + (1-\lambda_t)\theta\Omega'_{X_t} e\left[h_{t+1}, \cdot, \cdot\right]\right) u'_{C_t}}{Ah_{t+1}\left(\frac{\sigma+\Lambda_{t+1}}{\xi} + \phi + \left(1-\lambda_{t+1}\right)\theta\Omega'_{X_{t+1}} e\left[h_{t+2}, \cdot, \cdot\right]\right) u'_{C_{t+1}} - u'_{X_{t+1}}} \tag{24}$$

$$\frac{u'_{C_{t+1}}}{u'_{C_t}} = \frac{(1-\lambda_t)\theta h_t \Omega(X_t) e'_1\left[h_{t+1}, h_t, \overline{h}_t\right]}{\beta[1-(\frac{\sigma+\Lambda_{t+1}}{\xi}+\phi)X_{t+1} - \left(1-\lambda_{t+1}\right)\theta\Omega(X_{t+1}) e[h_{t+2}, h_{t+1}, \bar{h}_{t+1}]\left(1+\varepsilon_{h_{t+1}}^{e[\cdot, h_{t+1}, \cdot]}\right)} \tag{25}$$

By definition, an optimal economic policy has to make the competitive path identical to the first best path. To do so, the sequence $\{\lambda_t, \Lambda_t, T_t\}_{t=0}^{+\infty}$ has to make identical the sub systems $\{(20), (21)\}$ and $\{(24), (25)\}$ for each date t.

Optimality properties

Proposition

Given the parental (perfect) expectations on $\{\lambda_{t+1}, \Lambda_{t+1}, T_{t+1}\}_{t=0}^{t=+\infty}$ *, there exists a unique vector* $\{\lambda_t, \Lambda_t, T_t\}_{t=0}^{t=+\infty}$ *that is able to decentralize the first-best path. Given that* $\widehat{T}_t$ *is directly deduced from the government budget constraint the optimal economic policy is fully described as follows:*

$$\begin{aligned} \widehat{\lambda}_t &= -a_t\widehat{\lambda}_{t+1} + b_t\widehat{\Lambda}_{t+1} + d_t\left[F\left(X_t\right) - 1\right] - g_t \\ \widehat{\Lambda}_t &= -i_t\widehat{\lambda}_{t+1} + j_t\widehat{\Lambda}_{t+1} - k_t\left[F\left(X_t\right) - 1\right] - m_t \end{aligned}$$

Optimality properties

Once the Social Planner observes the parental expectations on the future values of the instruments, it is always possible to define a unique optimal economic policy.

At the steady state, the optimal economic policy is described by the set $\left\{\widehat{\lambda}, \frac{\widehat{\Lambda}}{\xi}\right\}$:

$$\widehat{\lambda} = \frac{-F(X)\varepsilon_3^e - \frac{\beta\varepsilon_L^f X}{Ah\Omega(X)\theta e(1-\beta F(X))}\frac{u}{u_C'} - [1-F(X)]\left[\frac{C}{Ah\Omega(X)\theta e} - \varepsilon_2^e\right]}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega}$$

$$\frac{\widehat{\Lambda}}{\xi} = -\frac{\beta\varepsilon_L^f}{[1-\beta F(X)]Ah}\frac{u}{u_C'}\frac{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega} - \frac{\Omega_X'\theta e\left[F(X)\varepsilon_3^e+(1-F(X))\left(\frac{C}{Ah}-\Omega(X)\theta h e_2'\right)\right]}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega}$$

Optimality properties

Interpretation 1: The purely Millian case $\left(f'\left(L_t\right)=0\right)$

In the Millian case, there is no difference between individual and social preferences. At the steady state, the optimal economic policy is described as follows:

$$\widehat{\lambda}=-\frac{\varepsilon_3^{\widehat{e}}}{1+\frac{\varepsilon_1^{\widehat{e}}}{\beta}+\varepsilon_2^{\widehat{e}}-\varepsilon_{\widehat{X}}^{\Omega}} \quad \frac{\widehat{\Lambda}}{\xi}=-\frac{\Omega'_{\widehat{X}}\theta\widehat{e}\varepsilon_3^{e}}{1+\frac{\varepsilon_1^{\widehat{e}}}{\beta}+\varepsilon_2^{\widehat{e}}-\varepsilon_{\widehat{X}}^{\Omega}} \quad \widehat{T}=\frac{A\widehat{h}\theta\widehat{e}\varepsilon_3^{\widehat{e}}\left(\varepsilon_{\widehat{X}}^{\Omega}-1\right)}{1+\frac{\varepsilon_1^{\widehat{e}}}{\beta}+\varepsilon_2^{\widehat{e}}-\varepsilon_{\widehat{X}}^{\Omega}} \tag{26}$$

Intuition: The subsidy on education decreases the total cost of a child, then agents tend to make too much children.

Optimality properties

Interpretation 2: The non-Millian case

$$\widehat{\lambda} = \frac{-F(X)\varepsilon_3^e - \frac{\beta\varepsilon_L^f X}{Ah\Omega(X)\theta e(1-\beta F(X))}\frac{u}{u'_C} - [1-F(X)]\left[\frac{C}{Ah\Omega(X)\theta e} - \varepsilon_2^e\right]}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega} \quad (27)$$

$$\frac{\widehat{\Lambda}}{\xi} = \frac{-\Omega'_X\theta eF(X)\varepsilon_3^e - \frac{u}{u'_C}\frac{\beta\varepsilon_L^f(1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e)}{[1-\beta F(X)]Ah} - \Omega'_X\theta e(1-F(X))\left(\frac{C}{Ah}-\Omega(X)\theta he'_2\right)}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega} \quad (28)$$

When $f(L) \neq 1$, the optimal tax transfer policy is not necessarily a subside on education and a tax on births.

a. The impact of the externality on education investment is the same as in the previous case

Interpretation 2: The non-Millian case

$$\widehat{\lambda} = \frac{-F(X)\varepsilon_3^e - \frac{\beta\varepsilon_L^f X}{Ah\Omega(X)\theta e(1-\beta F(X))}\frac{u}{u'_C} - [1-F(X)]\left[\frac{C}{Ah\Omega(X)\theta e} - \varepsilon_2^e\right]}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega} \quad (29)$$

$$\frac{\widehat{\Lambda}}{\xi} = \frac{-\Omega'_X\theta eF(X)\varepsilon_3^e - \frac{u}{u'_C}\frac{\beta\varepsilon_L^f(1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e)}{[1-\beta F(X)]Ah} - \Omega'_X\theta e(1-F(X))\left(\frac{C}{Ah}-\Omega(X)\theta he'_2\right)}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega} \quad (30)$$

When $f(L) \neq 1$, the optimal tax transfer policy is not necessarily a subside on education and a tax on births.

b. **Social value of population stock:** when $X > 1$ (increasing population), the Social Planner transfer welfare from present to future generations, educational investments of present generations have to be increased (positive effect on the optimal value of the subside). All other things being equal: positive impact on the optimal value of the tax on births because it decreases the total costs of children. The reverse is true when $X < 1$.

Optimality properties

Interpretation 2: The non-Millian case

$$\widehat{\lambda} = \frac{-F(X)\varepsilon_3^e - \frac{\beta\varepsilon_L^f X}{Ah\Omega(X)\theta e(1-\beta F(X))}\frac{u}{u'_C} - [1-F(X)]\left[\frac{C}{Ah\Omega(X)\theta e} - \varepsilon_2^e\right]}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega} \tag{31}$$

$$\frac{\widehat{\Lambda}}{\xi} = \frac{-\Omega'_X\theta e F(X)\varepsilon_3^e - \frac{u}{u'_C}\frac{\beta\varepsilon_L^f\left(1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e\right)}{[1-\beta F(X)]Ah} - \Omega'_X\theta e(1-F(X))\left(\frac{C}{Ah}-\Omega(X)\theta h e'_2\right)}{1+\frac{\varepsilon_1^e}{\beta}+\varepsilon_2^e-\varepsilon_X^\Omega} \tag{32}$$

When $f(L) \neq 1$, the optimal tax transfer policy is not necessarily a subside on education and a tax on births.

c. When $X > 1$, the optimal population growth rate may be high and the preference for larger generations too, competitive fertility rate is potentially too low, this has a negative impact on the tax rate on births.

- When $\frac{1}{\beta} > X > 1$, this effect does not dominate the previous one and so: $\lambda > 0$ and $\Lambda > 0$
- When $X > \frac{1}{\beta}$, this effect could be so high that $\lambda < 0$ and $\Lambda < 0$

Optimality properties

What about the two alternative models?

- Barro and Becker: $V_0 = \sum_{t=0}^{+\infty} \beta^t L_t^{1-\varepsilon} U(C_t)$
- Joy of giving: $V_t = u(c_t, n_t, h_{t+1})$

All the results hold unchanged...

Optimality properties

What did we learn here?

- Simple concepts of efficiency and optimality become useless when population is endogenous
- The most simple model of fertility deliver counter-intuitive if not disturbing results in terms of economic policy
- Obviously, fertility may be a source of positive externalities too...

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